

Chapter 2

International Student Achievement in Advanced Mathematics

Chapter 2 focuses on the TIMSS Advanced 2008 achievement results for students enrolled in advanced mathematics courses in the final year of secondary school in each of the participating countries. The chapter also addresses trends in mathematics achievement over time for participants in the previous TIMSS assessment at this level in 1995. Achievement differences by gender are also discussed.

Distribution of Advanced Mathematics Achievement in the Participating Countries

Exhibit 2.1 shows the distribution of student achievement in mathematics for the participants in TIMSS Advanced 2008, including the average (mean) scale score with its 95 percent confidence interval and the ranges in performance for the middle half of the students (25th to 75th percentiles), as well as the extremes (5th and 95th percentiles). Countries are listed in decreasing order of average scale score.

TIMSS Advanced 2008 used item response theory (IRT) methods to summarize the advanced mathematics achievement for each country on the TIMSS Advanced mathematics scale with a mean of 500 and



a standard deviation of 100.¹ The TIMSS Advanced mathematics scale for reporting the TIMSS Advanced 2008 results was established by rescaling the data from the 1995 TIMSS mathematics assessment of students in the final year of secondary school together with the mathematics data from the 2008 assessment using the scaling procedures currently used by TIMSS, and the methodology enables comparable trend measures from assessment to assessment.² That is, on the newly developed TIMSS Advanced mathematics scale, a score of 500 in advanced mathematics in 2008 is equivalent to a score of 500 in advanced mathematics in 1995.³ It should be noted, however, that achievement on the TIMSS Advanced mathematics scale cannot be described in absolute terms (as would be the case with all such scales developed using IRT technology), so these results cannot be directly compared to those for TIMSS Advanced physics found in Chapter 8. Comparisons between achievement in advanced mathematics and achievement in physics can only be made in terms of relative performance (higher or lower) among countries as well as between assessments.

Exhibit 2.1 shows that the 10 countries participating in the TIMSS Advanced 2008 mathematics assessment had considerable differences in their average achievement. At the top is a group of three countries—the Russian Federation, the Netherlands, and Lebanon. As shown by the symbol next to a participant’s average scale score indicating whether the average achievement is significantly higher (up arrow) or significantly lower (down arrow) than the scale average of 500, each of the three top-performing countries had average achievement higher than the international scale average of 500. The average scale scores for these three countries are relatively close to one another compared to the rest of the participating countries (ranging from 561 to 545), with each of the countries having average achievement

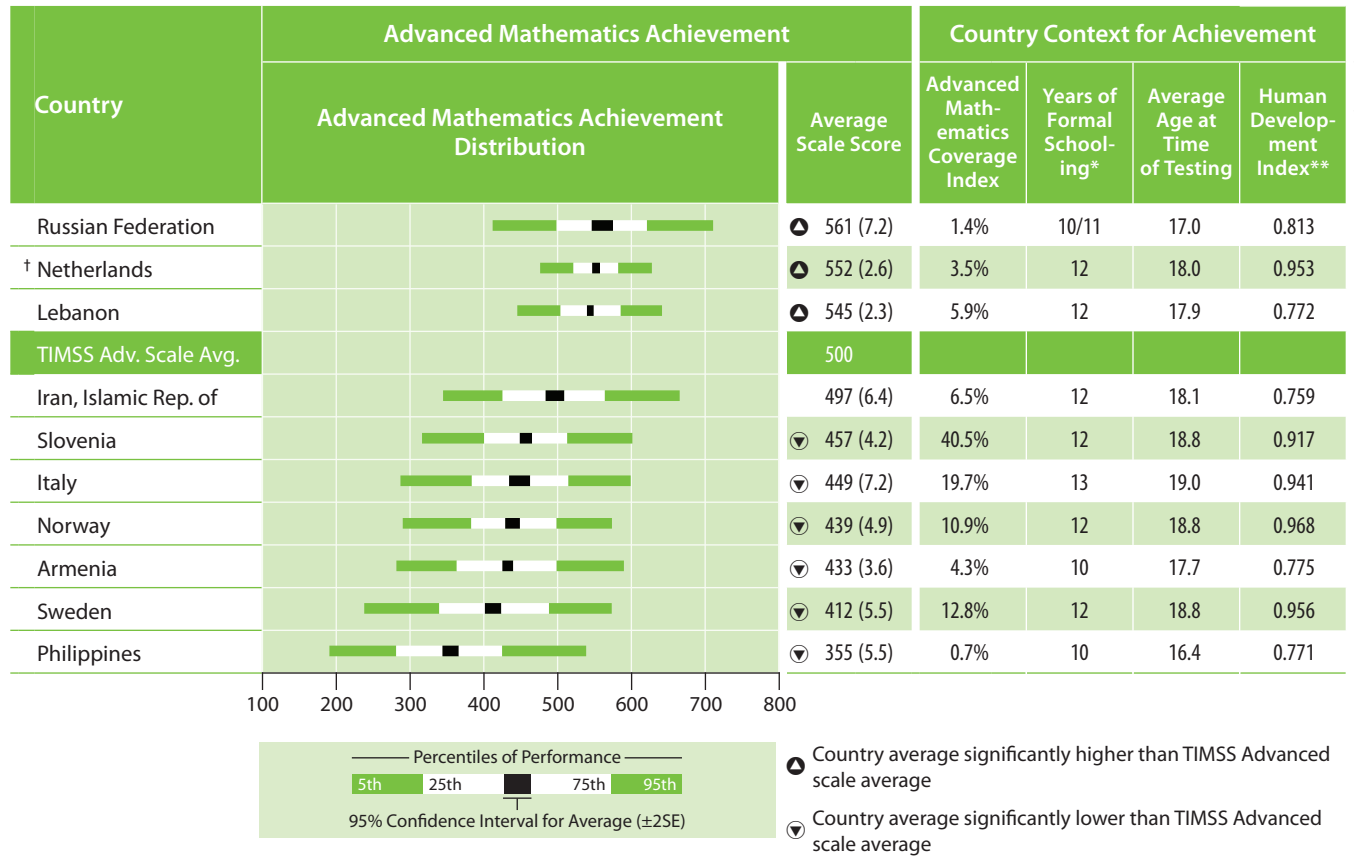
1 Given the matrix-sampling approach, the scaling process averages students’ responses in a way that accounts for differences in the difficulty of different subsets of items. It allows students’ performance to be summarized on a common metric even though individual students responded to different items in the advanced mathematics test.

2 Please see Appendix A for further information. A detailed description of the TIMSS Advanced 2008 scaling is provided in Foy, P., Galia, J., & Li, I. (2009). Scaling the data from the TIMSS Advanced 2008 mathematics and physics assessments. In A. Arora, P. Foy, M.O. Martin, and I.V.S. Mullis (Eds.), *TIMSS Advanced 2008 technical report*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

3 Because the rescaled 1995 data together with the 2008 data have been used in the analyses conducted for TIMSS Advanced 2008 and procedures differed from those used in 1995, the results from the 1995 data in this report cannot be compared directly with previous published 1995 achievement results.

Exhibit 2.1 TIMSS Advanced 2008 Distribution of Achievement in Advanced Mathematics

TIMSS Advanced 2008
Advanced Mathematics



SOURCE: IEA TIMSS Advanced 2008 ©

* Represents years of schooling counting from the first year of primary or basic education (first year of ISCED Level 1).
 ** Taken from United Nations Development Programme's *Human Development Report 2007/2008*, p.229-232.

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).
 () Standard errors appear in parentheses.

similar to next.⁴ However, there was a noticeable difference in average achievement between the Russian Federation with the highest average achievement and Lebanon (16 scale points), with the Netherlands in between the two. The Islamic Republic of Iran had average achievement below the top three countries and very close to the scale average (497).

The rest of the participating countries all had average achievement significantly below the scale average. The next cluster of countries in descending order by average achievement included Slovenia, Italy, Norway, and Armenia (457 to 433). These countries had average achievement that was similar from one country to the next adjacent country, although there was a significant difference between average achievement in Slovenia compared to Armenia (24 scale points). Next, Sweden's average achievement (412) was lower than that in Armenia (21 scale points). The Philippines, with an average scale score of 355, had the lowest average achievement.

The outer ends of the bar graphs in Exhibit 2.1 show the range of scores for a given country from the 5th to the 95th percentile. The Netherlands had the narrowest range of scores between the 5th and 95th percentiles, from a low of about 475 to a high of 625: about 1.5 standard deviations. Next was Lebanon with a somewhat wider range of about 200 points, or 2 standard deviations. The remaining 7 countries, including the highest scoring Russian Federation, had ranges close to or exceeding 300 scale points. That is, the range of scores within most countries exceeded the difference of 206 scale-score points across countries from the highest average achievement in the Russian Federation to the lowest in the Philippines.

As described in some detail in Chapter 1, there are many differences among the education systems of the countries that participated in TIMSS Advanced 2008. Because of these differences, there are a number of factors that need to be kept in mind in making

4 Taking into account the standard error provided in parentheses with each average scale score (mean achievement for the country), it can be said with 95 percent confidence that the corresponding value in the population falls between the sample estimate plus or minus two standard errors. Confidence intervals allow for an "eyeball" test of significance on whether the differences between the estimates (i.e., the means in this case) are statistically significant. If the confidence intervals of two estimates do not overlap, then differences in mean achievement are considered to be statistically significant. If the confidence intervals do overlap, then the estimates may or may not be statistically significantly different.

a comparison of the TIMSS Advanced achievement results. Exhibit 2.1 includes some of the basic information that needs to be taken into consideration. One essential factor to consider is that the number of years of schooling varied across countries (as described in more detail in Exhibit 1.1, and replicated here for ease of reference). Exhibit 2.1 shows the number of years of schooling completed in each country by the students who participated in TIMSS Advanced 2008 and their average age at the time of testing.

At the time of the TIMSS Advanced 2008 assessment, the students enrolled in advanced mathematics courses in their final year of secondary school were in their 12th year of formal schooling in six of the participating countries: the Netherlands, Lebanon, Iran, Slovenia, Norway, and Sweden. However, Italy reported 13 years; the Russian Federation, in the middle of implementing a reform to increase the number of years of schooling, reported some students with 10 years of schooling and some with 11 years; and Armenia and the Philippines reported 10 years. It should be noted that, as discussed in Chapter 1, a number of these countries have implemented reforms in the number of years of schooling since the TIMSS Advanced assessment or are in the process of doing so.

Because of differences among the years of schooling for these students in their final year as well as differences in age of entry to school and in promotion/retention policies, students' ages also varied across countries. The oldest students were in Slovenia, Italy, Norway, and Sweden, averaging from 18.8 to 19 years old. Students in the Netherlands, Lebanon, Iran, and Armenia were about a year younger, averaging from 17.7 to 18.1 years old. The students in the Russian Federation were even younger with an average age of 17, and, the students in the Philippines were the youngest, averaging 16.4 years of age. The three top-performing countries—the Russian Federation,

the Netherlands, and Lebanon—are not among those with the most years of schooling or the oldest students. However, the Philippines did have the youngest students and was one of the two countries with the fewest years of schooling.

Another important consideration in making comparisons in achievement is the variation in the proportion of students taking advanced mathematics in the final year of secondary school and included in the TIMSS Advanced 2008 mathematics population for the different countries. To quantify this difference among countries, TIMSS created the TIMSS Advanced 2008 coverage index presented in Exhibit 1.2. For ease of reference, the coverage index also is provided in Exhibit 2.1. For example, looking at the highest achieving countries, the Russian Federation included only 1.5 percent of its students in the TIMSS Advanced 2008 population from the possible population of all 17-year olds in the country. It seems rather surprising that such a highly selective population would produce such a wide range of scores. The Netherlands, on the other hand, included a slightly higher percentage of its age cohort of 18-year-old students in the assessed population (3.5%), and had a considerably narrower range of scale scores.

Exhibit 2.1 also contains each country's Human Development Index (HDI) value. The HDI was developed by the United Nations Development Programme, and is used in TIMSS and TIMSS Advanced 2008 to provide some context about the economic and educational development of the participants. The index has a minimum value of 0.0 and a maximum of 1.0. Countries with high values on the index (over 0.8 as defined by the UNDP) have long life expectancies, high levels of school enrollment and adult literacy, and a good standard of living, as measured by per capita Gross Domestic Product. Five of the TIMSS Advanced 2008 participants had index values over 0.9, including the Netherlands (0.953), Slovenia (0.917), Italy (0.941), Norway

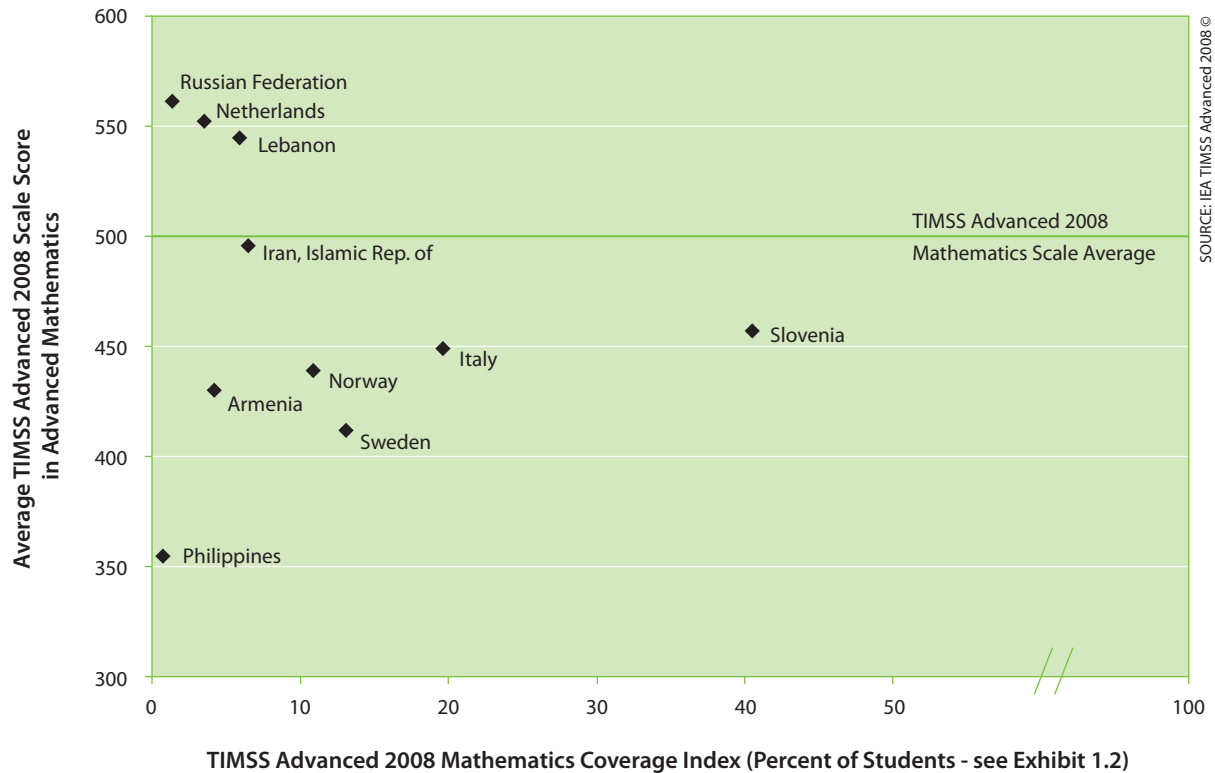
(0.968), and Sweden (0.956). With an index value of 0.813, just over the 0.8 borderline for the UNDP's high category, the Russian Federation also falls into the high category. However, four countries had index values in the 0.7 range and fall into the UNDP's medium category. Of the four countries, Armenia, Lebanon, and the Philippines had nearly identical HDIs (0.771–0.775), with that of Iran being only slightly lower (0.759). TIMSS results at the fourth and eighth grades have shown that while there is a positive relationship between having more country-wide resources and having higher average achievement in mathematics, the pattern is not always consistent⁵ and this appears to be the case for TIMSS Advanced 2008 mathematics. There is little consistency across the 10 countries in the relationship between a country's HDI value and average achievement in advanced mathematics for the specialized groups of students that participated in TIMSS Advanced 2008. For example, average achievement for the countries with HDIs over 0.9 ranged from a high of 552 in the Netherlands to a low of 412 in Sweden, the Russian Federation with a 0.813 HDI was the top-performing country, and achievement for the countries with HDIs in the 0.7 range spanned nearly 200 scale-score points from a high of 545 in Lebanon to a low of 355 in the Philippines.

Because of the importance of the proportion of the age cohort covered when considering how countries performed on the TIMSS Advanced 2008 mathematics assessment, Exhibit 2.2 presents average mathematics achievement in relation to the TIMSS Advanced 2008 coverage index for mathematics. In the graph, countries are arranged along the horizontal axis in ascending order of their TIMSS Advanced 2008 mathematics coverage index, from a low of 0.7 percent in the Philippines to a high of 41 percent in Slovenia. Countries are arranged along the vertical axis in ascending order of their average TIMSS Advanced 2008 scale scores for mathematics,

5 Mullis, I.V.S. & Martin, M.O. (2007). Lessons learned from TIMSS. In T. Loveless (Ed.), *Lessons learned from international assessments*. Washington, DC: Brookings Institution.

Exhibit 2.2 **Average Achievement in Advanced Mathematics by TIMSS Advanced 2008 Coverage Index for Advanced Mathematics**

TIMSS Advanced 2008
Advanced Mathematics



TIMSS Advanced 2008 Coverage Index for Advanced Mathematics

Country	Average Achievement	Coverage Index
Armenia	433	4.3%
Iran, Islamic Rep. of	497	6.5%
Italy	449	19.7%
Lebanon	545	5.9%
† Netherlands	552	3.5%
Norway	439	10.9%
Philippines	355	0.7%
Russian Federation	561	1.4%
Slovenia	457	40.5%
Sweden	412	12.8%

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

from a low of 355 in the Philippines to a high of 561 in the Russian Federation. The x -coordinate for the point corresponding to a given country, therefore, is the TIMSS Advanced coverage index for mathematics in that country, and the y -coordinate is the average scale score in mathematics. In general, the more to the right and the higher a country's point is on the graph, the better. And, correspondingly, the lower and the more to the left the point is, the more cause for concern there could be.

The results in Exhibit 2.2 reveal that none of the TIMSS Advanced participants were in the upper right hand corner, which would result from educating substantial proportions of students to high levels of achievement in advanced mathematics. Slovenia, with 41 percent of its population of final-year students assessed for TIMSS Advanced mathematics, is by far the farthest right followed by Italy with 20 percent. However, both had average mathematics achievement somewhat below the TIMSS scale average and in the middle of the participating countries. The three top performing countries—the Russian Federation, the Netherlands, and Lebanon—all included far smaller percentages of students than did Slovenia and Italy. However, looking at the three top-performing countries, each with somewhat successively lower achievement, it also can be seen that each also included a somewhat larger percentage of students in its TIMSS Advanced 2008 mathematics target population. Thus, taking the TIMSS Advanced 2008 Coverage Index into account, the results for the three countries could be considered even more similar than they appear to be looking only at average achievement.

Achievement on TIMSS Advanced 2008 Compared with Relative Achievement on TIMSS 2007

When the IEA began studying education internationally in the 1950s and 1960s, the populations compared often were to some degree comprised of elite students, especially at the secondary school level. That is, substantial proportions of students had dropped out of school and only the better students were continuing their schooling. Beyond that, most systems employed some type of tracking or streaming so that the better students received the more advanced education. However, as the years have gone by, more and more students in more and more countries are enrolled in basic education and also completing secondary education. Thus, recent international assessments conducted by TIMSS at the fourth and eighth grades⁶ provide results that pertain to the success countries are having in educating their entire school-aged populations. In contrast, TIMSS Advanced assesses the success countries have in educating a smaller proportion of select students to high levels of achievement on complicated content. Because all the TIMSS Advanced 2008 countries except the Philippines also participated in TIMSS 2007⁷ and the Philippine data are available from TIMSS 2003, it is interesting to make some comparisons among countries' relative standings in mathematics achievement internationally at the fourth and eighth grades compared to that for the advanced students in the final year of schooling (also keeping in mind the differences among the educational systems).

Exhibit 2.3 presents the average mathematics achievement in TIMSS 2007 and TIMSS Advanced for the TIMSS Advanced 2008 countries that participated in the mathematics assessment. For each assessment, countries are shown from highest to lowest average achievement, with symbols indicating statistically significant differences above or below the scale average.

6 Mullis, I.V.S., Martin, M.O., & Foy, P. (2008). *TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

7 All participated at the fourth grade except Lebanon and all at the eighth grade except the Netherlands. However, the Netherlands did in TIMSS 2003 at the eighth grade.

Exhibit 2.3 Average Mathematics Achievement at Fourth and Eighth Grades* and in the Final Year of Secondary School for the TIMSS Advanced 2008 Countries



TIMSS 2007 Mathematics – Fourth Grade		TIMSS 2007 Mathematics – Eighth Grade		TIMSS Advanced 2008 – Mathematics	
Country		Country		Country	
Russian Federation	544 (4.9) ▲	** Netherlands	536 (3.8) ▲	Russian Federation	561 (7.2) ▲
Netherlands	535 (2.1) ▲	Russian Federation	512 (4.1) ▲	Netherlands	552 (2.6) ▲
Italy	507 (3.1) ▲	Slovenia	501 (2.1)	Lebanon	545 (2.3) ▲
Sweden	503 (2.5)	TIMSS Scale Avg.	500	TIMSS Scale Avg.	500
Slovenia	502 (1.8)	Armenia	499 (3.5)	Iran, Islamic Rep. of	497 (6.4)
TIMSS Scale Avg.	500	Sweden	491 (2.3) ▼	Slovenia	457 (4.2) ▼
Armenia	500 (4.3)	Italy	480 (3.0) ▼	Italy	449 (7.2) ▼
Norway	473 (2.5) ▼	Norway	469 (2.0) ▼	Norway	439 (4.9) ▼
Iran, Islamic Rep. of	402 (4.1) ▼	Lebanon	449 (4.0) ▼	Armenia	433 (3.6) ▼
** Philippines	358 (7.9) ▼	Iran, Islamic Rep. of	403 (4.1) ▼	Sweden	412 (5.5) ▼
Lebanon	◊ ◊	** Philippines	378 (5.2) ▼	Philippines	355 (5.5) ▼

SOURCE: IEA TIMSS Advanced 2008 ©

- ▲ Country average significantly higher than TIMSS scale average
- ▼ Country average significantly lower than TIMSS scale average

* TIMSS 2007 data taken from Mullis, I.V.S., Martin, M.O., & Foy, P. (2008). *TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

** TIMSS 2003 data for the Netherlands at eighth grade and the Philippines at fourth and eighth grade taken from Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., & Chrostowski,

S.J. (2004). *TIMSS 2003 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College

() Standard errors appear in parentheses.

A diamond (◊) indicates the corresponding data are not available.

The Russian Federation performed above the scale average in all three assessments—fourth grade, eighth grade, and the final year of secondary school. It appears to be doing a good job of educating all of its students through lower secondary school as well as making it possible for a small percentage of elite students (1.4%) to reach a high level of excellence in mathematics by their final year of secondary school. Although the Russian Federation had the smallest coverage index, its students had 10 or 11 years of school (compared to 12 or 13) and were among the youngest (17 years old). It is especially noteworthy that all Russian students study mathematics and physics every year in lower secondary and upper secondary education, and the students assessed by TIMSS Advanced 2008 were having 6 hours or more of mathematics instruction per week. Similarly, the Netherlands demonstrated high achievement in TIMSS 2007 at the fourth grade, in TIMSS 2003 at the eighth grade, and for their mathematics specialists (3.5% of the age cohort) in TIMSS Advanced 2008. Its mathematics specialists were in a pre-university track and had studied 6 years of mathematics, the last three of which were part of an advanced program.

Norway also had a consistent relative standing across the three assessments, although performance was below the scale average in all three, including for their advanced students (10.9% of the age cohort). Since Norway has the highest HDI, these relatively low results cannot be explained by lack of resources. At the fourth and eighth grades, the TIMSS 2007 Norwegian results may partially be explained by the fact that those students started school at a younger age than in some countries and had a correspondingly less demanding curriculum in their early years of schooling. However, the Norwegian students in TIMSS Advanced are among the oldest in the assessment and according to their teachers have covered the TIMSS Advanced assessment topics to a large extent. The Philippines also had below

average results for the three populations of students, but its HDI value is among the lowest. Also, the Philippine students participating in TIMSS Advanced 2008 were among those with the fewest years of schooling, were the youngest, and according to their teachers had not been taught a considerable amount of the curriculum assessed.

Several countries had relatively lower achievement on TIMSS Advanced 2008 than on TIMSS 2007. Slovenia and Armenia performed at about the TIMSS scale average at the fourth and eighth grades, but below the scale average for TIMSS Advanced. Slovenia is a high HDI country and its students were in the 12th grade, averaging nearly 19 years old. However, it should be kept in mind that more than two fifths (41%) of the final-year students in Slovenia are being educated in advanced mathematics. Armenia's relative achievement for the students attending the special "physmat" schools (4.3%) in the final year of secondary school was relatively low, but Armenia is a middle HDI country and its TIMSS Advanced students were among those with the fewest years of schooling and less curriculum coverage, especially in calculus. Italy and Sweden performed close to the TIMSS scale average at the fourth grade, but below the TIMSS scale average at the eighth grade and also below the scale average on TIMSS Advanced 2008. Both of these countries have high HDI values and among the oldest students, but also relatively higher coverage indices with Sweden's advanced mathematics students comprising 13 percent of the age cohort and, in particular, Italy's advanced mathematics students comprising 20 percent of the age cohort.

Two countries, Lebanon and Iran, had relatively higher achievement on TIMSS Advanced 2008 than on TIMSS 2007. For both countries, the TIMSS Advanced 2008 students were in the 12th grade and just about 18 years old on average. Lebanon was one of the top-performing countries on TIMSS Advanced 2008, but this is in contrast

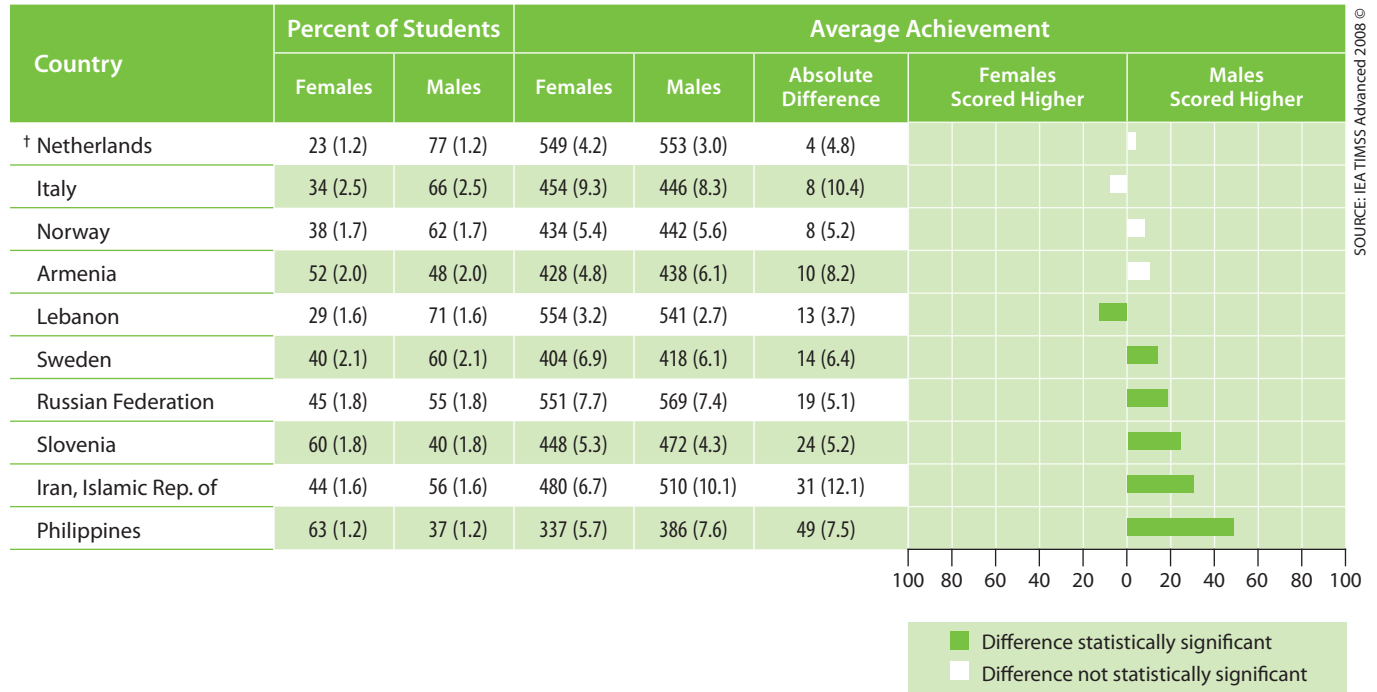
to its TIMSS 2007 performance, which was below the TIMSS scale average at the eighth grade. Similarly, Iran performed at about the TIMSS Advanced 2008 scale average, in contrast to its performance in TIMSS 2007 of approximately 100 scale points below the TIMSS scale average at both the fourth and eighth grades. These two countries are facing a number of challenges that have likely impacted their TIMSS results, including socioeconomic difficulties (medium category HDIs). Nevertheless, as evidenced by their TIMSS Advanced 2008 results, these countries have educated select groups of students (about 6%) to relatively high levels of achievement in mathematics internationally.

Gender Differences in Advanced Mathematics Achievement in the Participating Countries

Exhibit 2.4 shows the percentages of girls and boys enrolled in advanced mathematics in each of the participating countries and their differences in mathematics achievement on TIMSS Advanced 2008. It presents average achievement separately for females and males for the TIMSS Advanced 2008 countries, as well as the absolute difference between the two averages. The difference between the average achievement of females and males is shown in the graph by a bar indicating the amount of the difference, whether the direction of the difference was positive for females or males, and whether the difference is statistically significant (indicated by a darkened bar). Countries are shown in increasing order of the absolute difference in average achievement between females and males.

Armenia was the only country with equivalent percentages of female students (52%) and male students (48%) taking advanced courses in mathematics, although the Russian Federation and Iran had nearly equivalent percentages (about 45% female and 55% male). The greatest imbalance was in the Netherlands, where 77% of the students

Exhibit 2.4 TIMSS Advanced 2008 Average Achievement in Advanced Mathematics by Gender



† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

were male. Also, in Italy, Norway, Lebanon, and Sweden, from 60 to 66 percent of the students were male. In Slovenia and the Philippines, there was approximately a 60/40 split with the larger percentage of students being female.

In four countries, there was essentially no difference in average achievement in advanced mathematics between female students and male students. The four countries with equity in performance include the Netherlands, Italy, Norway, and Armenia. It can be noted that the Netherlands, the country with greatest imbalance in enrollment by gender, and Armenia, the country with least imbalance, are both included among the countries with no differences in average achievement by gender.

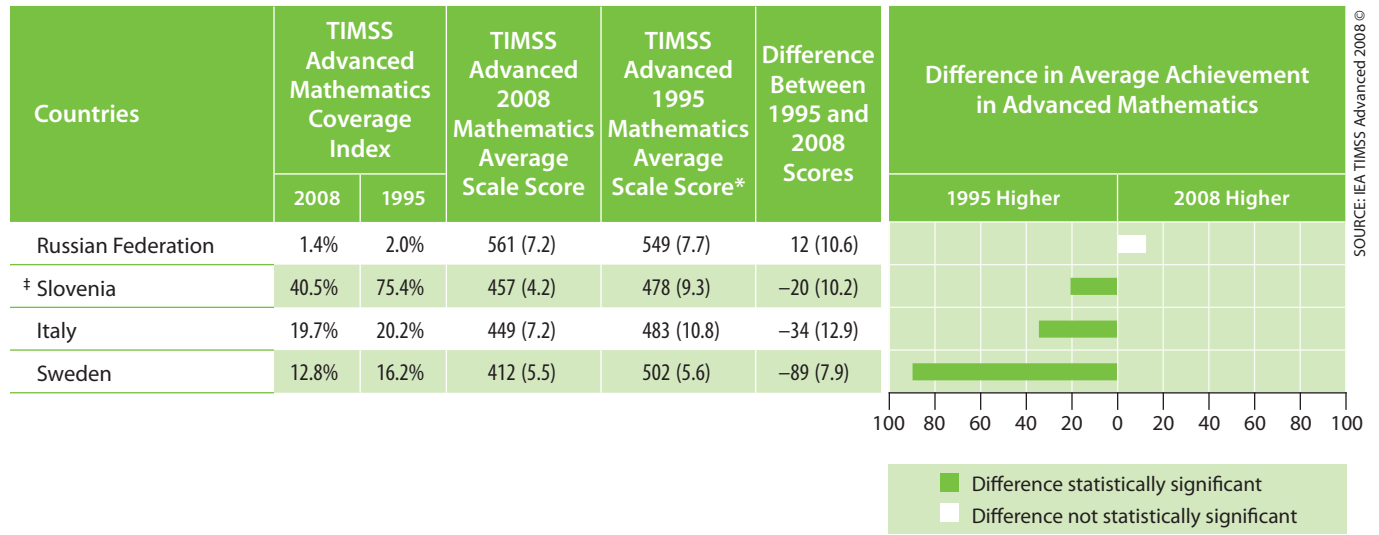
There were significant differences in achievement by gender in six of the participating countries, with the difference favoring males in five of them. Although females in Lebanon had significantly higher average scale scores than their male counterparts, male students had significantly higher average achievement in advanced mathematics in 5 of the 10 participating countries. In particular, the advantage for male students was rather large in the Philippines and Iran—almost 50 scale score points in the former, and about 30 in the latter.

Changes in Advanced Mathematics Achievement Between 1995 and 2008

Exhibit 2.5 displays changes in average advanced mathematics achievement for the four countries that participated in both the 1995 and 2008 cycles of this study, and these data are shown together with changes in the TIMSS Advanced coverage index. Coverage was comparable for Italy and the Russian Federation in both assessments, but there were changes for the other two trend countries. Coverage was considerably less in 2008 for Slovenia than it was in 1995, decreasing

Exhibit 2.5 Trends in Average Achievement in Advanced Mathematics

TIMSS Advanced 2008
Advanced Mathematics



SOURCE: IEA TIMSS Advanced 2008 ©

* To measure trends, the 1995 data were rescaled together with the 2008 data. Because procedures differed from those used in 1995, the achievement results for the 1995 assessment in this report cannot be compared directly with previously published 1995 achievement results.

‡ In 1995, did not satisfy guidelines for sample participation rates (see Appendix A).
 () Standard errors appear in parentheses.

Exhibit 2.6 Trends in Average Achievement in Advanced Mathematics by Gender

TIMSS Advanced 2008
Advanced Mathematics

Country	Females		Males		
	2008 Average Scale Score	1995 to 2008 Difference	2008 Average Scale Score	1995 to 2008 Difference	
Italy	454 (9.3)	-23 (15.7)	446 (8.3)	-41 (15.2)	▼
Russian Federation	551 (7.7)	25 (11.4) ▲	569 (7.4)	0 (11.3)	
‡ Slovenia	448 (5.3)	-21 (12.5)	472 (4.3)	-14 (11.9)	
Sweden	404 (6.9)	-88 (8.5) ▼	418 (6.1)	-88 (9.6) ▼	▼

SOURCE: IEA TIMSS Advanced 2008 ©

▲ 2008 average significantly higher than 1995
 ▼ 2008 average significantly lower than 1995

‡ In 1995, did not satisfy guidelines for sample participation rates (see Appendix A).
 () Standard errors appear in parentheses.

from about 75 to 41 percent, and coverage for Sweden also was reduced to some extent, from approximately 16 to 13 percent.

The participants are shown in the exhibit according to the difference between their average achievement in 1995 and 2008. In three of the four countries—Slovenia, Italy, and Sweden—average achievement in advanced mathematics declined significantly between the two assessments. Sweden showed the greatest average decline—almost 90 points. In the Russian Federation, average achievement in 2008 showed some signs of improvement but was not statistically different from that in 1995. The reasons underlying changes such as these in achievement over a substantial amount of time are difficult to pinpoint. For example, many cultural and educational factors could be involved, including changes in how the country organizes schooling, modifications in the advanced mathematics curriculum, and possibly changes in the characteristics and attitudes of the student population deciding to study advanced mathematics. Examining various hypotheses for the changes will take careful investigation and study.

Exhibit 2.6 shows changes in average achievement separately for females and males. Statistically significant decreases in achievement were found for male students in Italy and both groups in Sweden. It appears that the overall declines in achievement in advanced mathematics in Italy may be more related to greater declines by male students (41 points), on average, than by female students (23 points). In Sweden, both genders had equivalent decreases in average achievement (88 scale points). In Slovenia, the decrease in overall average achievement in 2008 compared to 1995 was related to non-statistically significant changes in a negative direction for both males and females.

Females in the Russian Federation had the only significant increase in average achievement between 1995 and 2008 (25 scale points). The

improvement by female students in the Russian Federation appears to underlie the indication of improvement overall, since males showed absolutely no difference in achievement between assessments.

Achievement Differences Across the TIMSS Advanced 2008 Mathematics Content and Cognitive Domains

As described in the *TIMSS Advanced 2008 Assessment Frameworks*,⁸ the advanced mathematics assessment was organized around two dimensions, a content dimension specifying the subject matter or content domains to be assessed in mathematics and a cognitive dimension specifying the thinking processes that students were deemed likely to use as they engaged with the content. Each item in the mathematics assessment was associated with one content domain and one cognitive domain, providing for both content-based and cognitive-oriented perspectives on student achievement in mathematics.

This section presents average student performance in the three content domains of the advanced mathematics framework: algebra, calculus, and geometry. Average performance also is presented for each of three cognitive domains: knowing, applying, and reasoning. Knowing refers to the student's knowledge base of mathematical facts, concepts, tools, and procedures. Applying focuses on the student's ability to apply knowledge and conceptual understanding in a problem situation. Reasoning goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems.

Students' performance across the three content domains and the three cognitive domains is summarized in Exhibit 2.7. The table shows the average percent correct for all of the advanced mathematics items for each country as well as within the six domains. Standard errors are shown in parentheses. This analysis by content and cognitive domains

8 Garden, R.A., Lie, S., Robitaille, D.F., Angell, C., Martin, M.O., Mullis, I.V.S., Foy, P., & Arora, A. (2006). *TIMSS Advanced 2008 Assessment Frameworks*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

uses average percent correct rather than average scale scores because there were insufficient items in all of the different domains to develop reliable scales. The countries are listed in alphabetical order.

In Armenia, students did relatively better in the algebra content domain than they did overall and relatively less well in calculus. The result in calculus is consistent with the reports that Armenia covered fewer of the TIMSS Advanced calculus topics than the other participating countries. In the cognitive domains, Armenian students did relatively better in the knowing domain than they did overall and less well in the applying domain. Iranian students and Italian students had similar achievement patterns across domains, demonstrating consistency with their overall average achievement in the content domains, but relatively higher average achievement on the knowing items and lower average achievement on the applying items. Dutch students also had consistent performance across the content domains, but had relatively higher average achievement in the reasoning domains and relatively lower average achievement in knowing and applying. Students in Lebanon performed relatively better in geometry and less well in algebra, and better in knowing and less well in applying and reasoning. Compared to their overall average achievement, students in Norway, the Philippines, and Slovenia demonstrated relative weakness in the calculus domain and relative strength in the geometry domain. For the Philippines and to a lesser extent Slovenia, this is consistent with teacher reports that they did not feel well prepared to teach some calculus topics and some calculus topics were not taught to sizeable percentages of students. Norway had consistent performance across the cognitive domains, whereas the Philippines had relative strength in knowing and relative weakness in applying. Slovenia's relative strength was in knowing and relative weakness in applying. Students in the Russian Federation did comparatively better in the content domain

Exhibit 2.7 **Average Percent Correct in the Advanced Mathematics Content and Cognitive Domains**



Country	Advanced Mathematics (71 Items)	Advanced Mathematics Content Domains			Advanced Mathematics Cognitive Domains		
		Algebra (25 Items)	Calculus (25 Items)	Geometry (21 Items)	Knowing (27 Items)	Applying (27 Items)	Reasoning (17 Items)
Armenia	32 (0.7)	37 (0.8) ▲	27 (0.6) ▼	33 (0.8)	39 (0.7) ▲	27 (0.8) ▼	31 (0.8)
Iran, Islamic Rep. of	43 (1.4)	45 (1.5)	41 (1.4)	44 (1.4)	52 (1.3) ▲	36 (1.4) ▼	42 (1.7)
Italy	35 (1.1)	33 (1.2)	36 (1.3)	36 (1.1)	40 (1.1) ▲	31 (1.2) ▼	33 (1.3)
Lebanon	53 (0.5)	51 (0.6) ▼	53 (0.6)	55 (0.5) ▲	65 (0.5) ▲	43 (0.6) ▼	51 (0.6) ▼
† Netherlands	54 (0.5)	55 (0.5)	53 (0.6)	53 (0.6)	51 (0.5) ▼	51 (0.6) ▼	63 (0.6) ▲
Norway	33 (0.7)	33 (0.8)	30 (0.7) ▼	37 (0.7) ▲	34 (0.7)	33 (0.7)	32 (0.8)
Philippines	24 (0.6)	24 (0.9)	19 (0.5) ▼	31 (0.6) ▲	28 (0.7) ▲	21 (0.7) ▼	24 (0.6)
Russian Federation	57 (1.6)	62 (1.6) ▲	53 (1.6)	56 (1.6)	59 (1.4)	56 (1.7)	56 (1.7)
Slovenia	36 (0.7)	38 (0.7)	32 (0.8) ▼	38 (0.9) ▲	41 (0.8) ▲	34 (0.8)	33 (0.7) ▼
Sweden	31 (0.7)	32 (0.9)	28 (0.8) ▼	32 (0.6)	32 (0.8)	28 (0.7) ▼	34 (0.8) ▲

SOURCE: IEA TIMSS Advanced 2008 ©

- ▲ Significantly higher than overall Advanced Mathematics percent correct
- ▼ Significantly lower than overall Advanced Mathematics percent correct

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

() Standard errors appear in parentheses. Because percents are rounded to the nearest whole numbers, some results may appear inconsistent.

of algebra than they did overall and otherwise their performance was consistent across both content and cognitive domains. Swedish students performed relatively less well in calculus but showed achievement in the other content domains consistent with their overall performance, which might be surprising considering that some of the geometry topics were not included in the curriculum. Across the cognitive domains, they showed relative weakness in applying and strength in reasoning.

Exhibit 2.8 presents the content and cognitive domain results by gender. The upper portion of the exhibit summarizes the results in the three content domains by gender; and the lower portion does the same for the three cognitive domains. Results for Italy show no significant differences in average achievement between females and males in any of the six content and cognitive domains. Also, Armenia, the Netherlands, and Norway had almost no differences in average achievement by gender; except males had higher average achievement than females in calculus and in applying in Armenia, higher average achievement in knowing in the Netherlands, and in reasoning in Norway.

As would be expected given the general advantage for males across the TIMSS Advanced assessment (see Exhibit 2.4), when there was a difference in achievement between genders, the male students typically had higher average achievement. In the Russian Federation, males had higher average achievement than females in geometry and in reasoning. In Sweden, males had higher average achievement than females in algebra and in reasoning.

Several countries had gender differences in most of the content areas. In the Philippines and in Slovenia, males had significantly higher average scores than females in all six areas—the three content domains and the three cognitive domains. In Iran, males had significantly higher average achievement than females in all except the knowing

Exhibit 2.8 **Average Percent Correct in the Advanced Mathematics Content and Cognitive Domains by Gender**

TIMSS Advanced 2008
Advanced Mathematics

Country	Average Percent Correct for Advanced Mathematics Content Domains					
	Algebra		Calculus		Geometry	
	Females	Males	Females	Males	Females	Males
Armenia	36 (0.9)	39 (1.4)	25 (0.8)	31 (0.9)	33 (1.0)	34 (1.5)
Iran, Islamic Rep. of	40 (1.7)	49 (2.3) ▲	37 (1.6)	44 (2.1) ▲	39 (1.6)	47 (2.2) ▲
Italy	32 (1.6)	34 (1.4)	37 (1.9)	35 (1.4)	35 (1.8)	36 (1.2)
Lebanon	53 (1.1) ▲	50 (0.7)	56 (1.0) ▲	53 (0.6)	56 (0.8) ▲	54 (0.7)
Netherlands	54 (1.0)	55 (0.6)	52 (1.2)	53 (0.7)	52 (1.3)	54 (0.6)
Norway	32 (0.9)	34 (0.9)	29 (0.7)	30 (0.9)	37 (0.8)	37 (0.8)
Philippines	22 (0.7)	29 (1.5) ▲	17 (0.5)	22 (0.9) ▲	29 (0.6)	34 (0.9) ▲
Russian Federation	60 (1.7)	64 (1.6)	51 (2.0)	54 (1.5)	54 (1.7)	59 (1.6) ▲
Slovenia	36 (0.8)	41 (1.1) ▲	31 (0.9)	34 (1.0) ▲	36 (1.0)	42 (1.1) ▲
Sweden	30 (0.9)	34 (1.2) ▲	27 (1.2)	28 (0.9)	31 (0.8)	33 (0.8)

SOURCE: IEA TIMSS Advanced 2008 ©

Country	Average Percent Correct for Advanced Mathematics Cognitive Domains					
	Knowing		Applying		Reasoning	
	Females	Males	Females	Males	Females	Males
Armenia	38 (0.9)	40 (1.4)	26 (0.9)	32 (1.5) ▲	31 (1.1)	32 (1.1)
Iran, Islamic Rep. of	50 (1.6)	53 (2.0)	31 (1.5)	40 (2.2) ▲	35 (1.8)	47 (2.6) ▲
Italy	40 (1.7)	40 (1.3)	31 (1.7)	31 (1.3)	32 (2.2)	34 (1.4)
Lebanon	68 (0.8) ▲	64 (0.5)	46 (1.0) ▲	42 (0.8)	50 (1.2)	51 (0.7)
† Netherlands	49 (1.0)	51 (0.6) ▲	50 (1.2)	51 (0.6)	62 (1.1)	63 (0.6)
Norway	35 (0.8)	34 (0.8)	33 (0.7)	33 (0.9)	29 (1.0)	34 (0.9) ▲
Philippines	26 (0.6)	31 (1.0) ▲	19 (0.5)	24 (1.1) ▲	22 (0.5)	28 (1.4) ▲
Russian Federation	58 (1.7)	60 (1.4)	54 (1.8)	57 (1.8)	52 (1.8)	60 (1.6) ▲
Slovenia	39 (0.9)	42 (1.0) ▲	32 (1.0)	36 (1.0) ▲	29 (0.8)	38 (1.1) ▲
Sweden	30 (1.0)	33 (0.9)	27 (0.8)	28 (0.9)	31 (1.1)	36 (1.1) ▲

▲ Significantly higher than other gender

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

() Standard errors appear in parentheses.

cognitive domain. In contrast, in Lebanon, females had higher average achievement than males in all except the reasoning domain.

Looking across countries, males had higher average achievement in each of the content domains in four countries compared to one country for females—Lebanon in each case. However, the countries where males had higher average achievement than females varied from content domain to content domain. Similarly, males had higher achievement than females in the knowing domain in three countries and in the applying domain in four countries compared to females having higher achievement in only one country—again, Lebanon in both cases. From the perspective of achieving gender equity in advanced mathematics achievement, perhaps of greatest concern is the finding that male students had higher average achievement than female students in the reasoning domain in 6 out of the 10 countries and females did not have higher average achievement than males in reasoning in any of the countries.

