

## CHAPTER

# 3

## Average Achievement in the Mathematics Content Areas

Chapter 3 presents results by the major content areas in mathematics to provide information about the possible effects of curricular variation on average achievement. Average performance is provided for five content areas: fractions and number sense; measurement; data representation, analysis, and probability; geometry; and algebra.



As delineated by the curriculum of the countries around the world and in the Benchmarking entities, mathematics contains a range of content areas (see Chapter 5 on curriculum). For example, almost all TIMSS 1999 countries and Benchmarking participants reported some elements of arithmetic as well as algebra and geometry in the eighth-grade mathematics curriculum. Since these content areas can differ in complexity, enter the curriculum at different times, receive varying degrees of emphasis, or even be taught as separate courses, Chapter 3 presents results by the major content areas in mathematics. For each Benchmarking entity, average achievement is shown for each content area and compared with the international average for that content area, and average achievement in the content areas is profiled in relation to overall mathematics achievement. Results are also provided by gender. These different perspectives are provided to identify the relative strengths and weaknesses of students in the different mathematics content areas as well as the possible effects of curricular variation on average achievement.

The TIMSS 1999 mathematics test for the eighth grade was designed to enable reporting by five content areas in accordance with the TIMSS mathematics framework. These areas, with their main topics, are:

- Fractions and number sense

*Includes whole numbers, fractions and decimals, integers, exponents, estimation and approximation, proportionality*

- Measurement

*Includes standard and non-standard units, common measures, perimeter, area, volume, estimation of measures*

- Data representation, analysis, and probability

*Includes representing and interpreting tables, charts, and graphs; range, mean; informal likelihood, simple numerical probability*

- Geometry

*Includes points, lines, planes, angles, visualization, triangles, polygons, circles, transformations, symmetry, congruence, similarity, constructions*

- Algebra


*Includes number patterns, representation of numerical situations, solving simple linear equations, operations with expressions, representations of relations and functions.*

## How Does Achievement Differ Across Mathematics Content Areas?

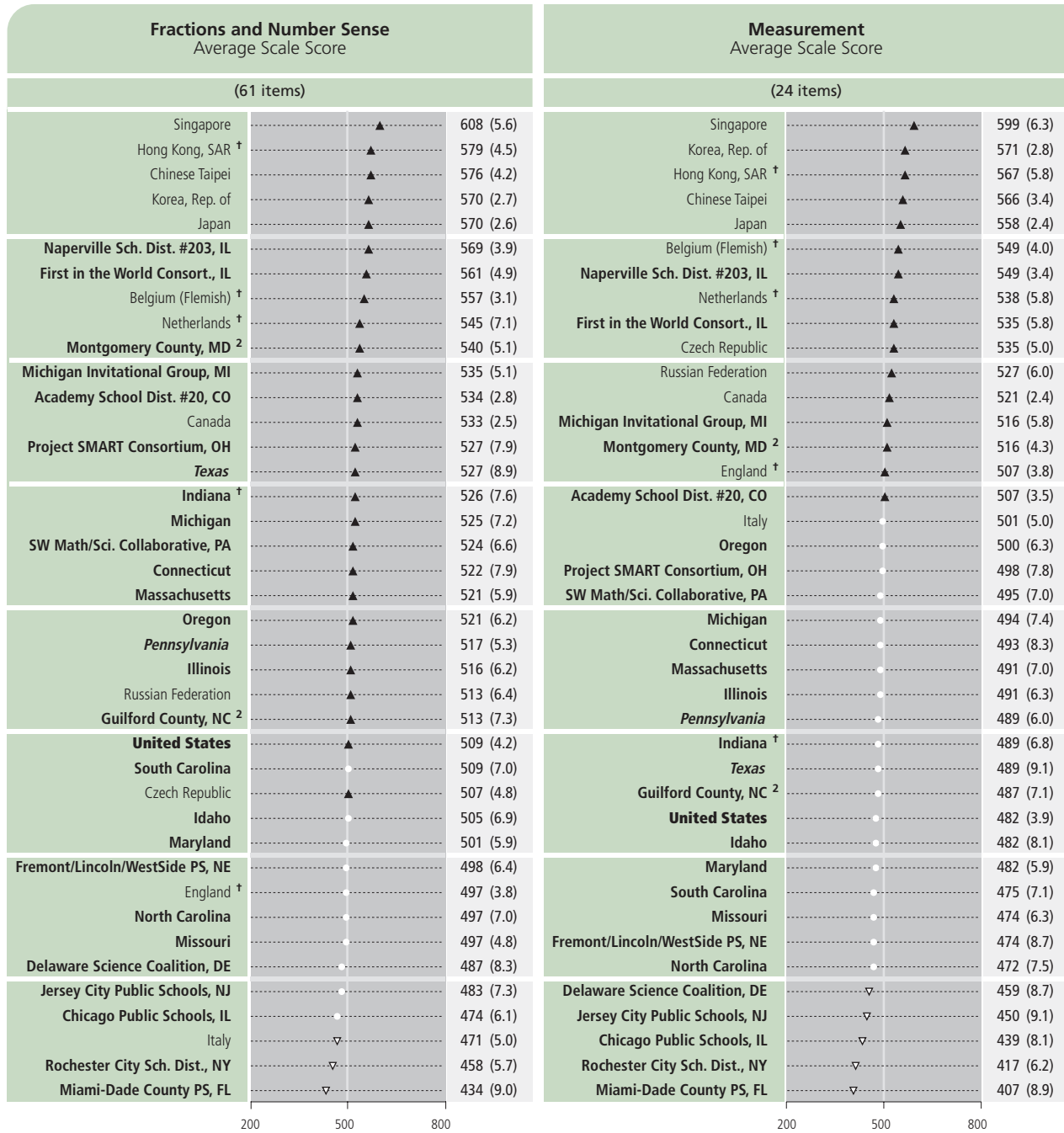
Exhibit 3.1 presents average achievement in each of the five mathematics content areas for the Benchmarking states, districts, and consortia. The Benchmarking jurisdictions as well as selected reference countries are displayed in decreasing order of achievement for each content area, and symbols indicate whether performance is statistically significantly above or below the international average for all of the countries that participated in TIMSS 1999. To allow comparison of the relative performance of each country in each content area, the international average for each content area was scaled to be 487, the same as the overall international average.

The six countries scoring highest in the overall mathematics assessment – Singapore, Korea, Chinese Taipei, Hong Kong, Japan, and Belgium (Flemish) – were also the highest-scoring countries (though not always in the same rank order) in each content area. Correspondingly, the Naperville School District and the First in World Consortium were the highest-scoring Benchmarking entities, performing significantly above the international average, and generally about the same as Belgium (Flemish), in each area.

In contrast to the consistent performance across content areas displayed by the highest-performing entities, performance varied substantially for some middle-performing entities, including the United States. The United States performed significantly above the international average in fractions and number sense; data representation, analysis, and probability; and algebra. In contrast, however, it performed similarly to the international average in measurement and geometry. The same pattern occurred in several of the Benchmarking jurisdictions, including the Project SMART Consortium, Texas, Indiana, Michigan, the Southwest Pennsylvania Math and Science Collaborative, Massachusetts, Oregon, and Guilford County. Montgomery County, the Michigan Invitational Group, and the Academy School District performed above the international average in measurement as well as in the three areas in which the U.S. did relatively well, but like the U.S. performed only at the international average in geometry. Although students in Pennsylvania and Illinois performed above the international average in fractions and number sense as well as in algebra, they performed similarly to the international average in the other three areas.



Exhibits B.1 through B.5 in Appendix B compare average achievement among individual entities for each of the content areas. The exhibits show whether or not the differences in average achievement between pairs of participating entities are statistically significant.



SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

▲ Participant average significantly higher than international average

● Participant average not significantly different from international average

▽ Participant average significantly lower than international average

Significance tests adjusted for multiple comparisons

**International Avg. (All Countries)** 487 (0.7)

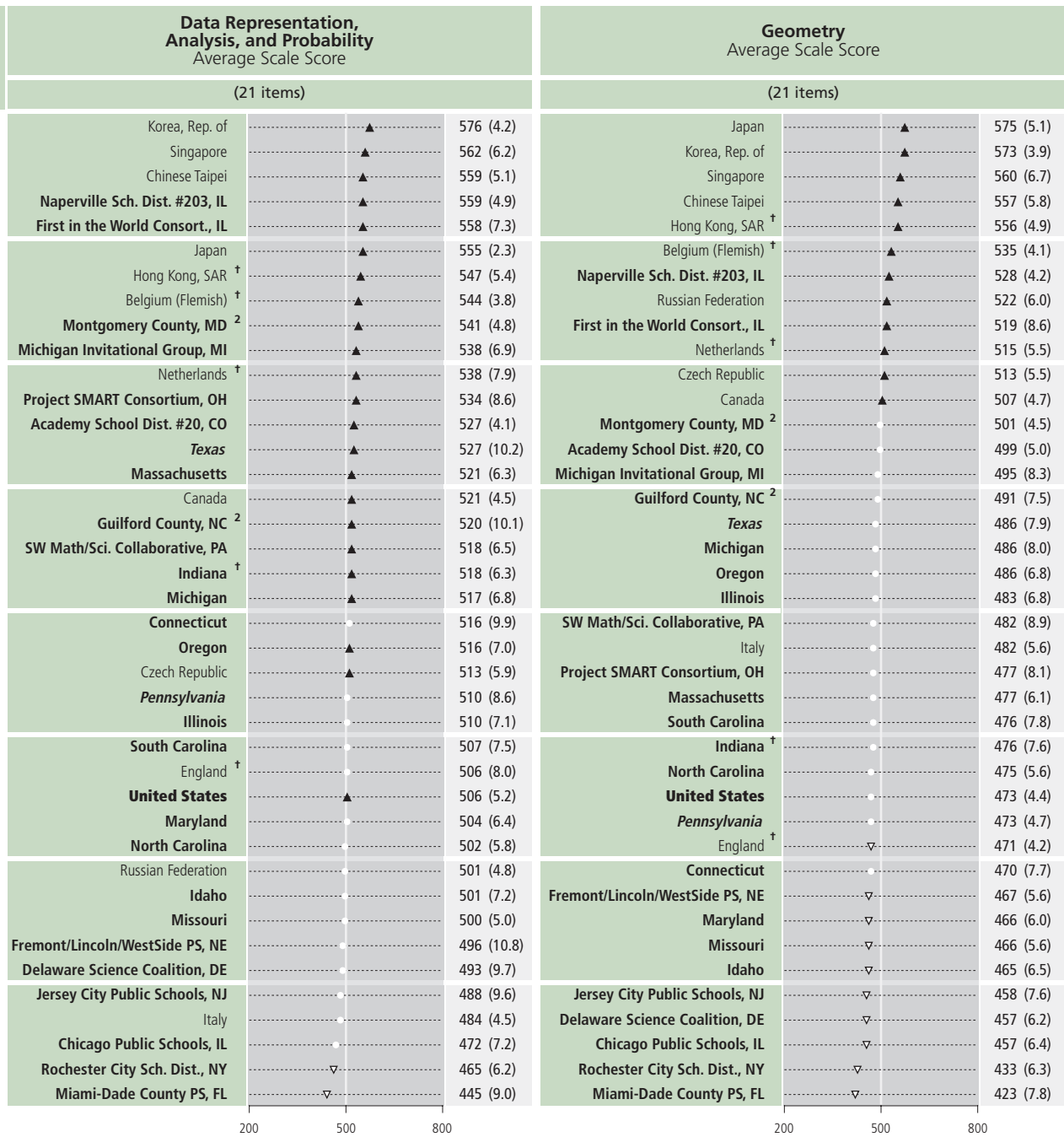
**International Avg. (All Countries)** 487 (0.7)

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

<sup>2</sup> National Defined Population covers less than 90% of National Desired Population (see Exhibit A.3).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.



SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

▲ Participant average significantly higher than international average

● Participant average not significantly different from international average

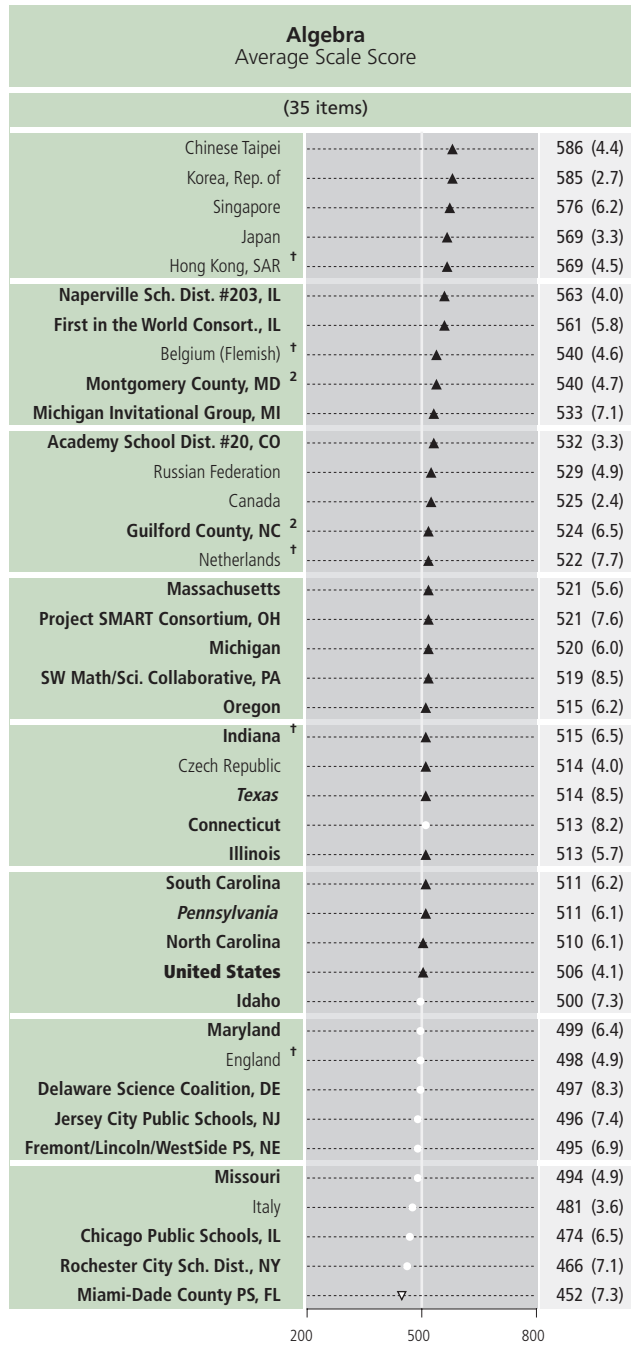
▽ Participant average significantly lower than international average

Significance tests adjusted for multiple comparisons

International Avg. (All Countries) 487 (0.7)

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SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

- ▲ Participant average significantly higher than international average
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Significance tests adjusted for multiple comparisons

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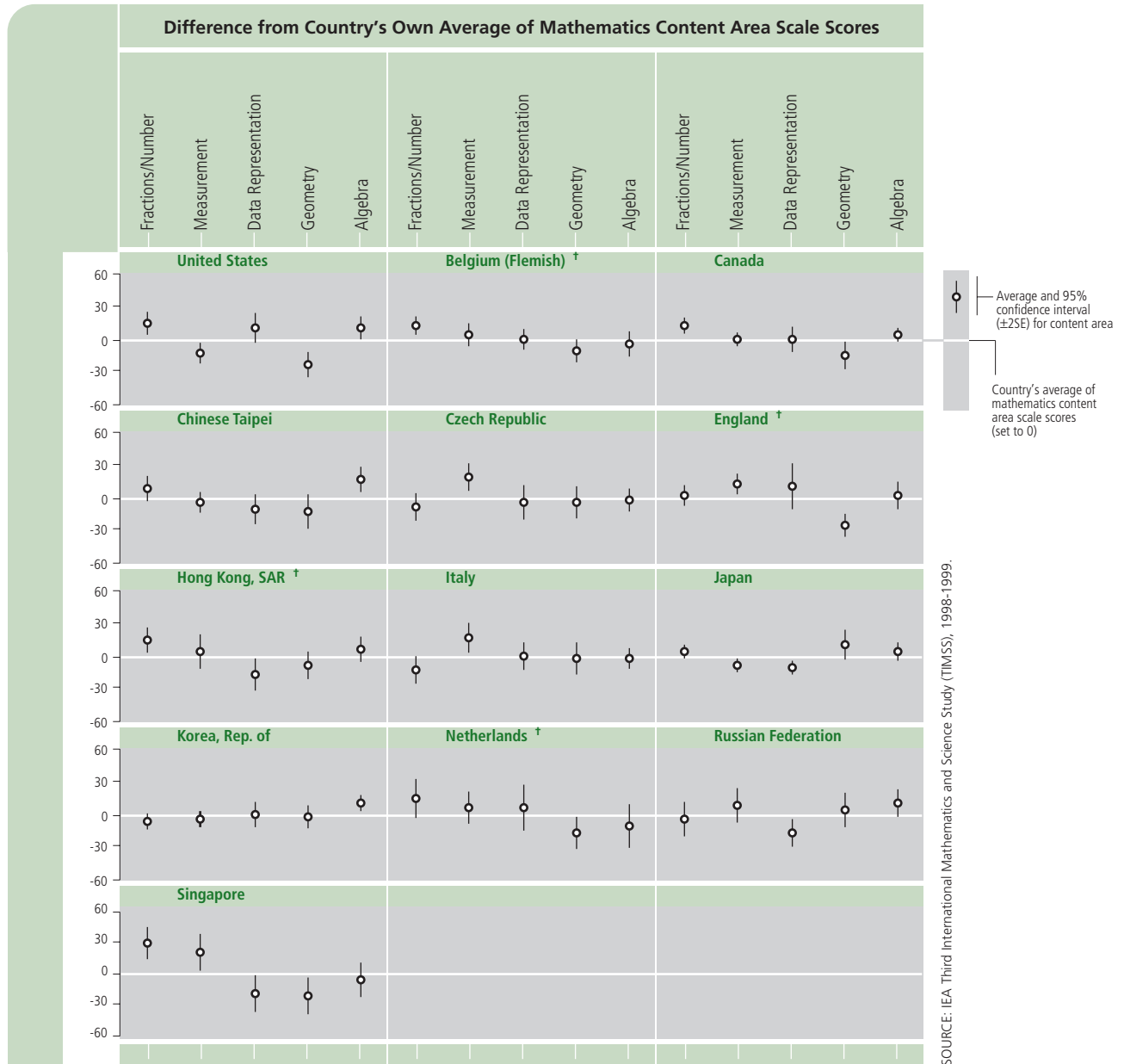


## In Which Content Areas Are Students Relatively Strong or Weak?

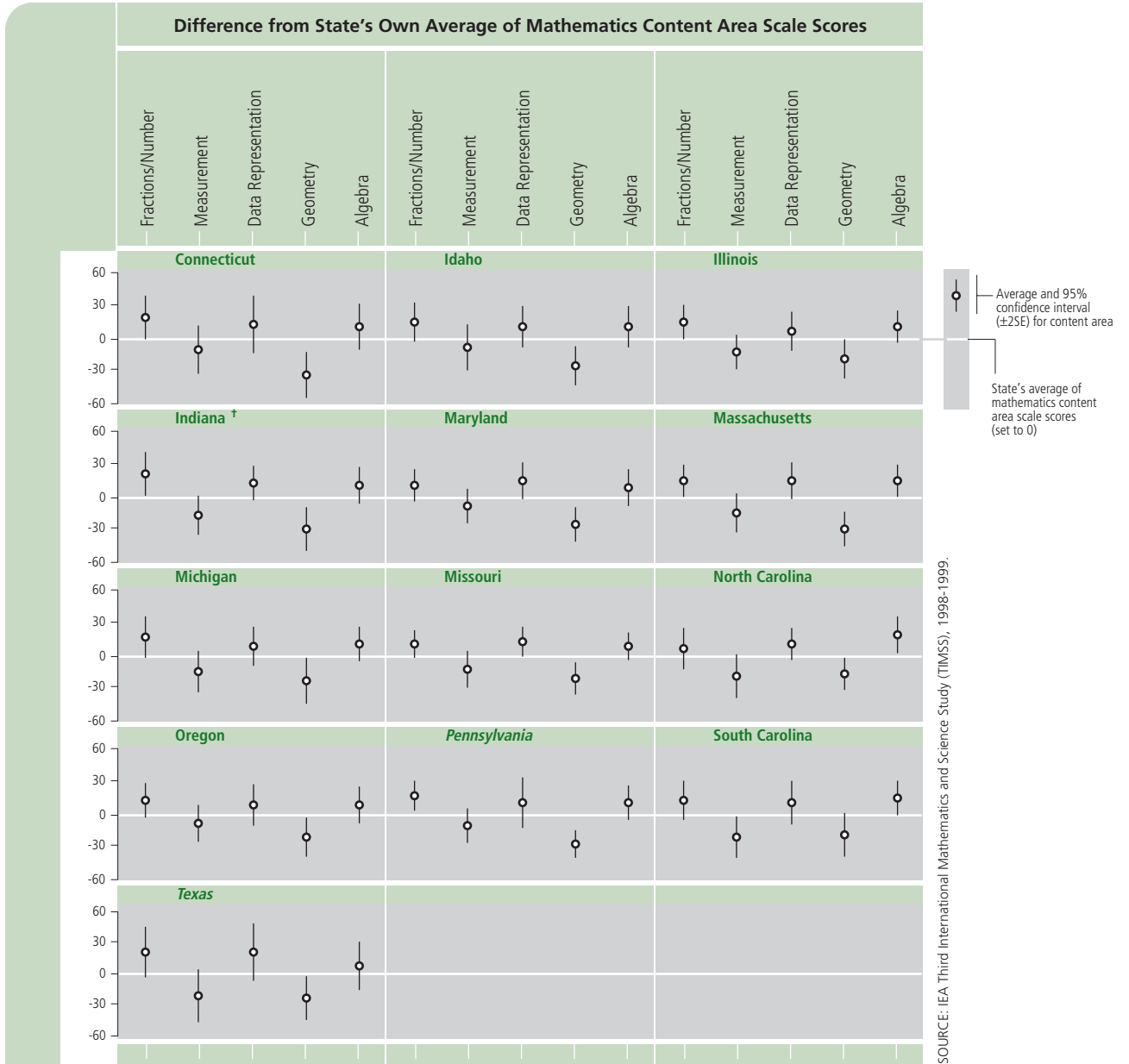
For purposes of comparison, Exhibit 3.2 profiles the relative performance in mathematics content areas within the comparison countries, while Exhibit 3.3 provides the corresponding information for the Benchmarking states and Exhibit 3.4 for the districts and consortia. These exhibits display the difference between average performance in each content area and average mathematics performance overall, highlighting any variation. The profiles reveal that as in the participating countries, students in many of the Benchmarking jurisdictions performed relatively better or worse in several content areas than they did overall. For example, students in all the Benchmarking entities generally followed the U.S. pattern of performing better than they did overall in fractions and number sense; data representation, analysis, and probability; and algebra, but less well in measurement and geometry.

In particular, a number of jurisdictions had relatively worse geometry performance, including Connecticut, Idaho, Indiana, Maryland, Massachusetts, Missouri, and Pennsylvania among the states. Districts and consortia with such results were the Academy School District, the Delaware Science Coalition, First in the World, the Fremont/Lincoln/Westside Public Schools, the Michigan Invitational Group, Montgomery County, Naperville, and Project SMART. Students' relatively low achievement in geometry is most likely related to less coverage of geometry topics in mathematics classrooms (see Chapter 5).

Among other notable findings, students in North and South Carolina did relatively well in algebra compared with their overall performance, and those in the Rochester City School District had particular difficulty in the area of measurement. Differences in relative performance may be related to one or more of a number of factors, such as emphases in intended curricula or widely used textbooks, strengths or weaknesses in curriculum implementation, and the grade level at which topics are introduced. For the Benchmarking entities, the patterns of relative strengths and weaknesses profiled in Exhibits 3.3 and 3.4 are sometimes reflected in strengths and weaknesses relative to other countries and the United States (shown in Exhibit 3.1).



† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).



States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

<sup>†</sup> Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).



<sup>2</sup> National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

## What Are the Gender Differences in Achievement for the Content Areas?

Exhibit 3.5 displays average achievement in mathematics content areas by gender for the Benchmarking entities as well as the comparison countries. The most striking feature of the exhibit is the very small number of statistically significant differences. There were no significant gender differences in average achievement in any Benchmarking jurisdiction, except that boys had higher average achievement than girls in fractions and number sense in Pennsylvania – for the Southwest Pennsylvania Math and Science Collaborative and for the state as a whole. Even though the United States had higher average achievement for boys than for girls in measurement, there were no significant differences in the Benchmarking entities.

An important stage of item selection for the TIMSS 1999 assessment was the examination of item statistics to detect items that differentiated between groups, including girls and boys, at the country level. Such items were scrutinized and retained when there was no apparent source of gender bias. It is therefore likely that the absence of significant gender differences in the averages for girls and boys in a country is due partly to a balance between items on which one or the other gender tends to perform better. It is also reasonable to assume that where significant differences do occur, they result from gender differences in one or more of the factors in student backgrounds and schooling that have consistently been found to affect achievement in mathematics.

In spite of there being few statistically significant differences in the average achievement of girls and boys in the content areas, it is interesting to look at the patterns of the differences. Consistent with the differences in the international averages, there was a strong tendency across the Benchmarking entities for boys to have higher average achievement than girls in fractions and number sense, measurement, and geometry. The results were more mixed in data representation, analysis, and probability and in algebra.

	Average Scale Scores for Mathematics Content Areas					
	Fractions and Number Sense		Measurement		Data Representation, Analysis, and Probability	
	Girls	Boys	Girls	Boys	Girls	Boys
<b>Countries</b>						
United States	505 (4.5)	514 (5.0)	475 (4.0)	489 (4.9) ▲	503 (7.0)	508 (6.3)
Belgium (Flemish) †	555 (6.0)	558 (7.7)	550 (6.5)	547 (8.2)	549 (6.7)	539 (8.8)
Canada	530 (2.4)	536 (3.4)	519 (4.6)	523 (4.4)	520 (5.2)	522 (6.6)
Chinese Taipei	574 (4.9)	579 (5.2)	563 (3.3)	569 (5.2)	557 (5.5)	561 (7.9)
Czech Republic	498 (5.7)	517 (6.1)	525 (6.1)	545 (6.6)	502 (7.0)	524 (6.9)
England †	487 (6.0)	507 (5.4)	500 (6.4)	515 (5.4)	498 (6.8)	513 (10.9)
Hong Kong, SAR †	579 (4.5)	578 (6.1)	567 (5.7)	567 (7.3)	546 (5.3)	548 (7.4)
Italy	463 (6.7)	479 (4.8)	494 (5.7)	508 (5.6)	483 (7.3)	484 (6.2)
Japan	563 (3.4)	576 (4.0)	556 (3.5)	559 (3.0)	552 (5.5)	559 (3.8)
Korea, Rep. of	566 (4.3)	573 (3.3)	567 (3.8)	575 (3.2)	574 (6.2)	579 (5.4)
Netherlands †	540 (7.9)	551 (7.5)	535 (7.5)	540 (6.2)	534 (10.3)	541 (8.3)
Russian Federation	510 (6.2)	516 (7.1)	524 (7.0)	529 (6.1)	502 (7.0)	501 (9.4)
Singapore	607 (6.2)	609 (6.8)	597 (7.3)	601 (9.0)	563 (6.8)	561 (8.8)
<b>States</b>						
Connecticut	514 (7.7)	530 (9.2)	484 (9.0)	503 (8.9)	512 (10.4)	520 (10.6)
Idaho	505 (6.9)	506 (8.5)	479 (8.7)	485 (8.4)	503 (8.3)	499 (8.8)
Illinois	511 (7.0)	522 (6.6)	489 (8.3)	494 (6.4)	506 (8.1)	514 (8.2)
Indiana †	518 (7.9)	534 (8.1)	481 (7.8)	497 (8.4)	514 (8.2)	522 (6.5)
Maryland	496 (6.4)	507 (6.4)	477 (6.7)	487 (5.8)	502 (7.7)	507 (8.5)
Massachusetts	516 (6.5)	526 (5.8)	486 (7.6)	497 (7.2)	520 (7.5)	523 (6.8)
Michigan	518 (7.2)	532 (7.7)	488 (7.7)	501 (8.5)	512 (7.9)	523 (7.3)
Missouri	494 (5.1)	500 (6.2)	470 (7.6)	478 (6.8)	499 (5.7)	500 (6.8)
North Carolina	492 (8.7)	502 (6.8)	471 (8.2)	473 (9.1)	504 (7.5)	499 (7.6)
Oregon	518 (6.9)	524 (7.0)	497 (7.3)	503 (7.4)	516 (8.4)	516 (11.5)
<i>Pennsylvania</i>	510 (5.8)	524 (5.6) ▲	482 (6.0)	497 (7.4)	508 (9.0)	513 (11.7)
South Carolina	506 (8.0)	512 (6.7)	473 (7.8)	477 (7.4)	508 (8.0)	506 (10.6)
<i>Texas</i>	526 (8.4)	527 (10.3)	482 (8.7)	495 (10.9)	530 (9.7)	524 (12.2)
<b>Districts and Consortia</b>						
Academy School Dist. #20, CO	530 (3.7)	539 (4.2)	500 (4.0)	514 (5.4)	530 (4.2)	524 (5.9)
Chicago Public Schools, IL	472 (6.3)	477 (6.5)	436 (9.8)	443 (7.3)	465 (9.5)	479 (7.9)
Delaware Science Coalition, DE	480 (8.9)	494 (10.7)	452 (10.1)	466 (10.8)	491 (10.2)	495 (13.0)
First in the World Consort., IL	556 (5.5)	566 (6.3)	530 (6.7)	540 (7.8)	548 (10.3)	568 (7.4)
Fremont/Lincoln/WestSide PS, NE	492 (7.1)	504 (8.0)	469 (9.0)	478 (10.2)	492 (12.5)	499 (11.7)
Guilford County, NC <sup>2</sup>	507 (7.8)	519 (8.2)	479 (9.4)	496 (7.6)	512 (10.8)	530 (11.2)
Jersey City Public Schools, NJ	479 (9.4)	486 (7.8)	446 (10.8)	454 (10.4)	487 (9.8)	489 (12.0)
Miami-Dade County PS, FL	432 (9.9)	437 (9.7)	405 (8.3)	410 (11.4)	444 (9.9)	446 (10.7)
Michigan Invitational Group, MI	538 (5.3)	533 (5.5)	512 (7.6)	520 (8.3)	547 (10.0)	530 (6.3)
Montgomery County, MD <sup>2</sup>	534 (6.5)	546 (6.3)	514 (5.9)	518 (6.9)	543 (6.9)	540 (5.6)
Naperville Sch. Dist. #203, IL	564 (4.9)	575 (4.1)	546 (5.0)	551 (4.5)	555 (7.7)	562 (9.3)
Project SMART Consortium, OH	524 (8.8)	530 (8.3)	496 (8.6)	499 (8.7)	539 (10.0)	529 (9.8)
Rochester City Sch. Dist., NY	451 (8.2)	465 (5.9)	405 (8.2)	431 (7.7)	465 (8.0)	464 (11.4)
SW Math/Sci. Collaborative, PA	517 (6.4)	531 (7.5) ▲	487 (6.9)	502 (9.0)	513 (7.8)	524 (7.8)
<b>International Avg. (All Countries)</b>	484 (0.9)	491 (0.9) ▲	483 (1.0)	491 (1.0) ▲	486 (1.1)	489 (1.1)

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

▲ Significantly higher than other gender

Significance tests adjusted for multiple comparisons

States in *italics* did not fully satisfy guidelines for sample participation rates (see Appendix A for details).

† Met guidelines for sample participation rates only after replacement schools were included (see Exhibit A.6).

<sup>2</sup> National Defined Population covers less than 90 percent of National Desired Population (see Exhibit A.3).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

	Average Scale Scores for Mathematics Content Areas			
	Geometry		Algebra	
	Girls	Boys	Girls	Boys
<b>Countries</b>				
United States	469 (5.5)	477 (5.1)	507 (4.3)	504 (4.6)
Belgium (Flemish) †	538 (6.9)	531 (9.1)	545 (6.8)	535 (8.8)
Canada	511 (6.5)	503 (4.9)	526 (3.7)	524 (5.2)
Chinese Taipei	555 (7.1)	560 (6.8)	585 (4.5)	588 (6.1)
Czech Republic	506 (7.6)	520 (4.9)	513 (3.9)	516 (6.7)
England †	467 (4.8)	474 (6.7)	493 (6.0)	502 (5.1)
Hong Kong, SAR †	558 (6.1)	554 (6.4)	570 (4.8)	568 (5.6)
Italy	476 (8.6)	489 (5.1)	481 (5.4)	481 (4.0)
Japan	572 (5.8)	578 (5.8)	568 (4.2)	571 (3.6)
Korea, Rep. of	569 (7.3)	578 (4.8)	585 (3.7)	585 (3.9)
Netherlands †	516 (7.0)	515 (5.2)	522 (9.3)	522 (7.4)
Russian Federation	518 (7.2)	526 (7.4)	533 (5.7)	524 (6.3)
Singapore	556 (9.2)	565 (6.5)	578 (6.7)	574 (7.9)
<b>States</b>				
Connecticut	465 (10.5)	475 (8.8)	510 (8.4)	516 (8.9)
Idaho	462 (9.3)	468 (7.1)	504 (7.0)	496 (8.5)
Illinois	479 (8.5)	487 (9.5)	514 (7.6)	511 (5.2)
Indiana †	471 (8.9)	481 (8.0)	516 (6.7)	514 (7.0)
Maryland	462 (5.6)	471 (7.8)	499 (7.4)	500 (7.4)
Massachusetts	475 (6.0)	478 (7.1)	522 (6.0)	521 (6.2)
Michigan	480 (7.0)	493 (10.8)	517 (6.6)	523 (6.6)
Missouri	464 (7.3)	468 (8.8)	495 (6.0)	493 (5.4)
North Carolina	473 (8.3)	478 (7.3)	512 (6.4)	507 (6.8)
Oregon	485 (8.9)	487 (6.9)	522 (6.4)	509 (7.1)
Pennsylvania	466 (5.9)	479 (5.5)	512 (7.2)	510 (7.1)
South Carolina	474 (9.8)	479 (9.1)	514 (6.4)	508 (7.0)
Texas	484 (7.3)	489 (9.8)	514 (9.0)	514 (9.0)
<b>Districts and Consortia</b>				
Academy School Dist. #20, CO	495 (6.5)	504 (5.5)	534 (3.6)	531 (5.4)
Chicago Public Schools, IL	457 (5.8)	456 (9.4)	475 (6.9)	473 (7.7)
Delaware Science Coalition, DE	456 (6.3)	458 (9.1)	495 (8.5)	499 (10.0)
First in the World Consort., IL	519 (7.2)	518 (12.5)	561 (7.6)	560 (6.3)
Fremont/Lincoln/WestSide PS, NE	461 (6.6)	473 (8.0)	496 (8.2)	495 (8.7)
Guilford County, NC <sup>2</sup>	487 (8.2)	495 (9.1)	522 (7.5)	527 (6.5)
Jersey City Public Schools, NJ	454 (10.3)	462 (7.2)	498 (8.0)	494 (7.8)
Miami-Dade County PS, FL	420 (8.8)	425 (9.3)	457 (8.1)	448 (7.7)
Michigan Invitational Group, MI	500 (8.9)	489 (10.2)	540 (6.6)	525 (8.8)
Montgomery County, MD <sup>2</sup>	500 (8.0)	502 (5.1)	542 (5.3)	537 (6.5)
Naperville Sch. Dist. #203, IL	522 (7.3)	534 (7.4)	561 (3.7)	565 (5.4)
Project SMART Consortium, OH	470 (9.7)	484 (10.9)	524 (7.0)	518 (9.3)
Rochester City Sch. Dist., NY	427 (10.7)	438 (9.5)	466 (8.5)	467 (8.0)
SW Math/Sci. Collaborative, PA	476 (9.3)	489 (9.9)	515 (8.9)	523 (8.7)
<b>International Avg. (All Countries)</b>	485 (1.2)	489 (1.1)	489 (0.9)	485 (0.9)

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1998-1999.

▲ Significantly higher than other gender

Significance tests adjusted for multiple comparisons

