Overview

Students should be educated to recognize mathematics as an immense achievement of humanity, and to appreciate its nature. Nevertheless, learning mathematics for its own sake is probably not the most compelling reason for universal inclusion of mathematics in school curricula. Prime reasons for having mathematics as a fundamental part of schooling include the increasing awareness that effectiveness as a citizen and success in a workplace are greatly enhanced by knowing and, more important, being able to use mathematics. The number of vocations that demand a high level of proficiency in the use of mathematics, or mathematical modes of thinking, has burgeoned with the advance of technology, and with modern management methods.

This chapter contains the framework for the TIMSS 2007 mathematics assessments at the fourth and eighth grades. At both grades, the mathematics assessment framework for TIMSS 2007 is organized around two dimensions, a content dimension specifying the domains or subject matter to be assessed within mathematics (for example, number, algebra, geometry, and data and chance at the eighth grade) and a cognitive dimension specifying the domains or thinking processes to be assessed (that is, knowing, applying, and reasoning). The cognitive domains describe the sets of behaviors expected of students as they engage with the mathematics content.

The content and cognitive domains are the foundation of the TIMSS 2007 fourth- and eighth-grade assessments. The content domains differ for the fourth and eighth grades, reflecting the nature and difficulty of the mathematics widely taught at each grade. There is more emphasis on number at the fourth grade than at the eighth grade. At the eighth grade, two of the four content domains are geometry and algebra, but since geometry and algebra generally are not taught as a formal subjects in primary school, the domain assessed at the fourth grade focuses on geometric shapes and measures and introductory algebra concepts are included as part of number. At the fourth grade, the domain pertaining to data focuses on reading and displaying data whereas at eighth grade it

includes more emphasis on interpretation of data and the fundamentals of probability (called "chance").

The cognitive domains are the same for both grades, encompassing a range of cognitive processes involved in working mathematically and solving problems right through the primary and middle school years.

Exhibit 2 shows the target percentages of testing time devoted to each content and cognitive domain for both the fourth- and eighth-grade assessments.

Exhibit 2: **Target percentages of the TIMSS 2007 Mathematics Assessment Devoted to Content and Cognitive Domains at Fourth and Eighth Grades**

Fourth-Grade Content Domains	Percentages	
Number	50%	
Geometric Shapes and Measures	35%	
Data Display	15%	
Eighth-Grade Content Domains	**Percentages**	
Number	30%	
Algebra	30%	
Geometry	20%	
Data and Chance	20%	
Cognitive Domains	**Percentages**	
	Fourth Grade	**Eighth Grade**
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

Each content domain has several topic areas (i.e., *number* at eighth grade is further categorized by whole numbers; fractions and decimals; integers; and ratio, proportion, and percent). Each topic area is presented as a list of objectives covered in many participating countries, at either fourth grade or eighth grade as appropriate.

The content and cognitive domains for the mathematics assessment are discussed in detail in the following sections. The content domains for the fourth grade are presented first, followed by those for

Mathematics Framework: Overview

the eighth grade. The cognitive domains, applicable to both grades, then follow. Example mathematics items and tasks are presented in Appendix B.

Mathematics Content Domains – Fourth Grade

The content domains described in the mathematics framework for the fourth grade and the target percentages of testing time devoted to each are shown below in Exhibit 3.

**Exhibit 3:   Target Percentages of the TIMSS 2007 Mathematics
Assessment Devoted to Content Domains
at Fourth Grade**

Fourth-Grade Content Domains	Percentages
Number	50%
Geometric Shapes and Measures	35%
Data Display	15%

The content domains define the specific mathematics subject matter covered by the TIMSS 2007 assessment at fourth grade. Each content domain has several topic areas; each one is presented as a list of objectives covered in the mathematics curriculum in the majority of participating countries. These grade specific objectives are written in terms of student understandings or abilities that items aligned with these objectives are designed to elicit. The following sections describe each of the mathematics content domains at fourth grade.

Number

The *number* content domain for the fourth grade includes understanding of place value, ways of representing numbers, and the relationships between numbers. At the fourth grade, students should have developed number sense and computational fluency, understand the meanings of operations and how they relate to one another, and be able to use numbers and operations (i.e. add, subtract, multiply, and divide) to solve problems. They should be familiar with a range of number

patterns, exploring the relationships between the numbers which are in the pattern or are used to derive it.

The number content domain consists of understandings and skills related to four topic areas:

- Whole numbers

- Fractions and decimals

- Number sentences

- Patterns and relationships

Since whole numbers provide the easiest introduction to operations with numbers that are basic to the development of mathematics, working with whole numbers is the foundation of mathematics in the primary school. The TIMSS 2007 content framework reflects this. Most children learn to count at a young age and can solve simple addition, subtraction, multiplication, and division problems during the first few years of school. Fourth-grade students should be able to compute with whole numbers of reasonable size, estimate sums, differences, products, and quotients, and use computation to solve problems.

Students also should be using their grasp of number to understand the relationships between units of measurement and to convert from one unit to another. Such relationships should include the multiples of 10 found in the metric system of measurement and other familiar ones such as the relationships between seconds, minutes, hours and days.

At the fourth grade, pre-algebraic concepts and skills are also part of the TIMSS assessment. The focus is on the type of understanding, which is built upon later to develop more formal, algebraic thinking. Understandings related to simple equations – in the form of number sentences – and to number patterns are included.

Students should be working with number sentences and finding missing numbers in them, working towards the idea of finding a value for an unknown, and using number sentences to model simple situations involving one of the four operations. They should be exploring number patterns, investigating the relationships between their terms and the finding or using the rules that generate them.

In the area of common fractions and decimal fractions, the emphasis is on representation of fractions and understanding what quantities the symbols represent. At the fourth grade, students should be able to compare familiar fractions and decimals.

Number: Whole Numbers

1. Represent whole numbers using words, diagrams, or symbols.
2. Demonstrate knowledge of place value, including recognizing and writing numbers in expanded form.
3. Compare and order whole numbers.
4. Know the four operations (+, −, ×, ÷) and compute with whole numbers.
5. Recognize multiples and factors of numbers; read weight and temperature scales marked in multiples.
6. Estimate computations by approximating the numbers involved.
7. Solve problems, including those set in real life contexts (for example, measurement and money problems).
8. Solve problems involving proportions.

Number: Fractions and Decimals

1. Recognize fractions as parts of unit wholes, parts of a collection, locations on number lines, and divisions of whole numbers.
2. Represent fractions using words, numbers, or models.
3. Identify equivalent fractions; compare and order fractions.
4. Add and subtract simple fractions.
5. Show understanding of decimal place value including recognizing and writing decimals using words and numbers.
6. Add and subtract decimals.
7. Solve problems involving simple fractions or decimals.

Note: Fourth-grade fractions items will involve denominators of 2, 3, 4, 5, 8, or 10. Fourth-grade decimals items will involve decimals to tenths and/or hundredths.

Mathematics Framework: Fourth-Grade Content Domains

Number: Number Sentences with Whole Numbers

1. Find the missing number or operation in a number sentence (e.g., if 17 + __ = 29, what number would go in the blank to make the number sentence true?).

2. Model simple situations involving unknowns with expressions or number sentences.

Number: Patterns and Relationships

1. Extend patterns and find missing terms in them.

2. Describe relationships between adjacent terms in a sequence or between the sequence number of the term and the term.

3. Generate pairs of whole numbers following a given rule (e.g., multiply the first number by 3 and add 2 to get the second number).

4. Write or select a rule for a relationship given some pairs of whole numbers satisfying the relationship.

Geometric Shapes and Measures

The *geometric shapes and measures* domain includes properties of geometrical figures such as lengths of sides, sizes of angles, areas, and volumes. Students should be able to identify and analyze the properties and characteristics of lines, angles, and a variety of geometric figures, including two- and three-dimensional shapes, and to provide explanations based on geometric relationships. This domain includes understanding informal coordinate systems and using spatial visualization skills to relate between two- and three-dimensional representations of the same shape.

The three topic areas in geometric shapes and measures are:

• Lines and angles

• Two- and three-dimensional shapes

• Location and movement

Spatial sense is integral to the study and assessment of geometry. At the fourth grade, students will be asked to describe, visualize, and draw a variety of geometric figures, including angles, lines, triangles, quadrilaterals, and other polygons. Students should be able to make and decompose compound shapes. They should be able to recognize line symmetry, draw symmetrical figures, and describe rotations and reflections.

At the fourth grade, appropriate performances expected of students include the use of instruments and tools to measure physical attributes, including length, area, volume, and angle. Knowledge about which units to use in particular contexts should underly their measurement skills. Students at this grade are also expected to use approximation and estimation, and simple formulas, to calculate areas and perimeters of squares and rectangles.

Geometric Shapes and Measures: Lines and Angles

1. Measure and estimate lengths.
2. Identify and draw parallel and perpendicular lines.
3. Compare angles by size and draw angles (e.g., a right angle, angles larger or smaller than a right angle).

Geometric Shapes and Measures: Two- and Three-dimensional Shapes

1. Identify common geometric shapes.
2. Know, describe, and use elementary properties of geometric figures.
3. Classify and compare geometric figures, (e.g., by shape, size or properties).
4. Recognize relationships between three-dimensional shapes and their two-dimensional representations.
5. Calculate areas and perimeters of squares and rectangles of given dimensions
6. Determine and estimate areas and volumes (e.g., by covering with a given shape or by recognizing that area is conserved).

Geometric Shapes and Measures: Location and Movement

1. Use informal coordinate systems to locate points in a plane.

2. Recognize and draw figures with line symmetry.

3. Recognize and draw reflections and rotations of figures.

Data Display

The *data display* content domain includes reading and interpreting displays of data. It also includes understanding how to organize data that have been collected and how to display it in graphs and charts that will be useful in answering the questions that prompted the data collection. Students should be able to compare characteristics of data and to draw conclusions based on data displays.

The data content domain consists of the following major topic areas:

• Reading and interpreting

• Organizing and representing

At the fourth grade, students should be able to read various data displays. Students also can engage in simple data-gathering plans or work with data that have been gathered by others. They should be developing skills in representing data and recognizing a variety of forms of data display.

Data Display: Reading and Interpreting

1. Read data from tables, pictographs, bar graphs, and pie charts.

2. Compare information from related data sets (e.g., given data or representations of data on the favorite flavors of ice cream in four or more classes, identify the class with chocolate as the most popular flavor).

3. Use information from data displays to answer questions that go beyond directly reading the data displayed (e.g., combine data, perform computations based on the data, draw conclusions, and make predictions).

Data Display: Organizing and Representing

1. Compare and match different representations of the same data.

2. Organize and display data using tables, pictographs, and bar graphs.

Mathematics Content Domains – Eighth Grade

The content domains described in the mathematics framework for the eighth grade and the target percentages of testing time devoted to each are shown below in Exhibit 4.

Exhibit 4:   Target Percentages of the TIMSS 2007 Mathematics Assessment Devoted to Content Domains at Eighth Grade

Eighth-Grade Content Domains	Percentages
Number	30%
Algebra	30%
Geometry	20%
Data and Chance	20%

The content domains define the specific mathematics subject matter covered by the TIMSS 2007 assessment at eighth grade. Each content domain has several topic areas; each one is presented as a list of objectives covered in the mathematics curriculum in the majority of participating countries. These grade specific objectives are written in terms of student understandings or abilities that items aligned with these objectives are designed to elicit. Sometimes the wording of objectives is similar or identical for fourth and eighth grades. In these instances, the progression in learning between the two grades is established by the difficulty of the items used. The following sections describe each of the mathematics content domains at eighth grade.

Number

The *number* content domain includes understanding of numbers, ways of representing numbers, relationships among numbers, and number systems.

At the eighth grade, students should have developed number sense and computational fluency, understand the meanings of operations and how they relate to one another, and be able to use numbers and operations to solve problems.

The number content domain consists of understandings and skills related to:

- Whole numbers

- Fractions and decimals

- Integers

- Ratio, proportion, and percent

The emphasis within computation is on fractions and decimals rather than on whole numbers. Within fractions and decimals, the emphasis is on representation and translation between forms, understanding what quantities the symbols represent, computation, and problem solving. By the eighth grade, students should be able to move flexibly among equivalent fractions, decimals, and percents using a range of strategies.

Eighth-grade students should have extended their mathematical understanding from whole numbers to integers, including order and magnitude as well as operations with integers. Students' should also be able to work with percents and proportions and use proportional reasoning to solve problems.

The problems students will be asked to solve include both the routine and the non-routine, those set in everyday contexts and those where mathematics itself is the context. Some problems involve computation with a range of measures and units of measurement.

Number: Whole Numbers

1. Demonstrate knowledge of place value and of the four operations.
2. Find and use multiples or factors of numbers, read scales, and identify prime numbers.

3. Use the principles of commutativity, associativity, and distributivity.

4. Evaluate powers of numbers, and square roots of perfect squares to 144.

5. Solve problems by computing, estimating, or approximating.

Number: Fractions and Decimals

1. Compare and order fractions and decimals.

2. Demonstrate knowledge of place value for decimals.

3. Represent decimals and fractions and operations with decimals and fractions using models (e.g., number lines); identify and use such representations.

4. Recognize and write equivalent fractions.

5. Convert between fractions and decimals.

6. Compute with fractions and decimals.

7. Solve problems by computing, estimating, and approximating.

Number: Integers

1. Represent, compare, order, and compute with integers.

2. Solve problems using integers.

Number: Ratio, Proportion, and Percent

1. Identify and find equivalent ratios; express ratios.

2. Divide a quantity in a given ratio.

3. Convert between percents and fractions or decimals.

4. Solve problems involving percents and proportions.

Algebra

While functional relationships and their uses for modeling and problem solving are of prime interest, it is also important to assess how well the supporting knowledge and skills have been learned. The *algebra* content domain includes recognizing and extending patterns, using algebraic symbols to represent mathematical situations, and developing fluency in producing equivalent expressions and solving linear equations.

The major topic areas in algebra are:

- Patterns

- Algebraic expressions

- Equations/formulas and functions

Algebraic concepts are relatively formalized by this grade, and students should have developed an understanding of linear relationships and the concept of variable. Students at this level are expected to use and simplify algebraic formulas, solve linear equations, inequalities, pairs of simultaneous equations involving two variables, and use a range of functions. They should be able to solve real-world problems using algebraic models and to explain relationships involving algebraic concepts.

Algebra: Patterns

1. Extend numeric, algebraic, and geometric patterns or sequences using numbers, words, symbols, or diagrams; find missing terms.

2. Generalize pattern relationships in a sequence, or between adjacent terms, or between the sequence number of the term and the term, using numbers, words, or algebraic expressions.

Algebra: Algebraic Expressions

1. Find sums, products, and powers of expressions containing variables.

2. Evaluate expressions for given numeric values of the variable(s).

3. Simplify or compare algebraic expressions to determine equivalence.

4. Model situations using expressions.

Algebra: Equations/Formulas and Functions

1. Evaluate equations/formulas given values of the variables.

2. Indicate whether a value (or values) satisfies a given equation/formula.

3. Solve simple linear equations and inequalities, and simultaneous (two variables) equations.

4. Recognize and write linear equations, inequalities, simultaneous equations, or functions that model given situations.

5. Recognize and generate equivalent representations of functions as ordered pairs, tables, graphs, or words.

6. Solve problems using equations/formulas and functions.

Geometry

Eighth-grade students should be able to analyze the properties and characteristics of a variety of two and three-dimensional geometric figures, including lengths of sides and sizes of angles, and to provide explanations based on geometric relationships. They should be able to apply the Pythagorean theorem to solve problems. The focus should be on using geometric properties and their relationships.

Alongside their appreciation of geometric properties and relationships, students also should be competent in geometric measurement, using measuring instruments accurately, estimating where appropriate, and selecting and using formulas for perimeters, areas, and volumes. The *geometry* content area also includes understanding coordinate representations and using spatial visualization skills to move between two- and three-dimensional shapes and their representations. Students should be able to use symmetry and apply transformation to analyze mathematical situations.

The three topic areas in geometry are:

• Geometric shapes

• Geometric measurement

• Location and movement

Spatial sense is integral to the study and assessment of geometry. The cognitive range extends from making drawings and constructions to mathematical reasoning about combinations of shapes and transformations. Students will be asked to describe, visualize, draw, and construct a variety of geometric figures, including angles, lines, triangles, quadrilaterals, and other polygons. Students should be able to combine, decompose, and analyze compound shapes. By this grade, they should be able to interpret or create top or side views of objects and use their understanding of similarity and congruence to solve problems.

Students should be able to use the Cartesian plane to locate points and lines. They should be able to recognize line symmetry and draw symmetrical figures. They should understand and be able to describe rotations, translations, and reflections in mathematical terms (e.g., center, direction, and angle).

As students progress through school, using proportional thinking in geometric contexts is important, as is making some initial links between geometry and algebra. Students should be able to solve problems using geometric models and explain relationships involving geometric concepts.

Geometry: Geometric Shapes

1. Classify angles as acute, right, straight, obtuse, and reflex (more than 180˚); draw such angles.

2. Know and use the relationships for angles at a point, angles on a line, vertically opposite angles, angles associated with a transversal cutting parallel lines, angle bisection, and perpendicularity.

3. Recall and use geometric properties of geometric shapes: triangles, quadrilaterals, and other common polygons.

4. Construct or draw triangles and rectangles of given dimensions.

5. Identify congruent triangles, quadrilaterals and their corresponding measures.

6. Identify similar triangles and recall their properties.

7. Recognize relationships between three-dimensional shapes and their two-dimensional representations, (e.g., nets or two-dimensional views of three-dimensional objects).

8. Use Pythagorean theorem (not proof) to solve problems.

9. Apply geometric properties to solve problems.

Note: Eighth-grade geometric shapes items will involve circles, the following triangles – scalene, isosceles, equilateral, and right-angled; the following quadrilaterals – scalene, trapezoid, parallelogram, rectangle, rhombus, and square; as well as other polygons including pentagon, hexagon, octagon, and decagon.

Geometry: Geometric Measurement

1. Measure, draw, and estimate the size of given angles.

2. Measure, draw, and estimate the length of lines, perimeters, areas and volumes.

3. Select and use appropriate measurement formulas for perimeters, circumferences, areas of circles, surface areas, and volumes.

4. Find measures of irregular or compound areas (e.g., by covering with grids or dissecting and rearranging pieces).

Geometry: Location and Movement

1. Use ordered pairs, equations, intercepts, intersections, and gradient to locate points and lines in the Cartesian plane.

2. Recognize and use line and rotational symmetry for two-dimensional shapes, e.g. to draw symmetrical figures.

3. Recognize, or demonstrate by sketching, translation, reflection, and rotation.

Data and Chance

The *data and chance* content domain includes knowing how to organize data that have been collected by oneself or others and how to display data in graphs and charts that will be useful in answering questions that prompted the data collection. This content domain includes understanding issues related to misinterpretation of data.

The data and chance content domain consists of the following three major topic areas:

- Data organization and representation

- Data interpretation

- Chance

Students can engage in simple data-gathering plans or work with data that have been gathered by others or generated by simulations. They should understand what various numbers, symbols, and points mean in data displays. For example, they should recognize that some numbers represent the values of the data and others represent the frequency with which those values occur. Students should develop skill in representing their data, often using bar graphs, tables, or line graphs. They should be able to recognize and compare the relative merits of various types of displays.

Students should be able to describe and compare characteristics of data (shape, spread, and central tendency), and draw conclusions based on data displays. Students should be able to identify trends in data, make predictions based on data, and evaluate the reasonableness of interpretations.

Eighth-grade students' appreciation of chance (elementary probability) should include being able to designate the occurrence of familiar events as certain; as having greater, equal, or less likelihood; or as impossible, and should extend to using data from experiments or knowledge of equally likely outcomes to predict the chance of a given outcome.

Data and Chance: Data Organization and Representation

1. Read data from tables, pictographs, bar graphs, pie charts, and line graphs.

2. Organize and display data using tables, pictographs, bar graphs, pie charts, and line graphs.

3. Compare and match different representations of the same data.

Data and Chance: Data Interpretation

1. Identify, calculate and compare characteristics of data sets, including mean, median, range, and shape of distribution (in general terms).

2. Use and interpret data sets to answer questions and solve problems (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points).

3. Recognize and describe approaches to organizing and displaying data that could lead to misinterpretation (e.g., inappropriate grouping and misleading or distorted scales).

Data and Chance: Chance

1. Judge the chance of an outcome as certain, more likely, equally likely, less likely, or impossible.

2. Use data from experiments to predict the chances of future outcomes.

3. Given a context, use the chances of a particular outcome to solve problems; determine the chances of possible outcomes (e.g., a particular face has a one-sixth chance of being on top after dropping a number cube).

Guidelines for Calculator Use

Although technology in the form of calculators and computers can help students learn mathematics, it should not be used to replace basic

understanding and competencies. Like any teaching tool, calculators need to be used appropriately, and policies for their use differ across the TIMSS countries. Also, the availability of calculators varies widely. It would not be equitable to require calculator use when students in some countries may never have used them. Similarly, however, it is not equitable to deprive students of the use of a familiar tool.

After considerable debate on the issue, TIMSS 2003 introduced calculator use in the eighth-grade mathematics assessment. For newly developed items, calculators were not required, but were permitted if participating countries wanted to allow their students to use them. Based on a study conducted as part of TIMSS 2003 where the same items were given before the break when calculators were not permitted and in the session after the break when calculators were allowed, it was found that even without specifically planning nearly all the TIMSS mathematics items could be answered just as easily without the use of a calculator. That is, performance was not significantly different with or without a calculator for all except five items. Also, of the students who had calculators (63 percent), the vast majority (47 percent) reported that although they had calculators, they used them very little or not at all.

Based on the experience in TIMSS 2003, for TIMSS 2007 eighth-grade students will be permitted to use calculators for the entire assessment. As with previous TIMSS assessments, fourth-grade students will not be permitted to use calculators.

The aim of the TIMSS guidelines for calculator use is to give students the best opportunity to operate in settings that mirror their classroom experience. Thus, if students are used to having calculators for their classroom activities, then the country should encourage students to use them during the assessment. On the other hand, if students are not used to having calculators or are not permitted to use them in their daily mathematics lessons, then the country need not permit their use. In developing the new assessment materials, every effort will be made to ensure that the test questions do not advantage or disadvantage students either way – with or without calculators.

Mathematics Cognitive Domain – Fourth and Eighth Grades

To respond correctly to TIMSS test items students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. Describing these skills plays a crucial role in the development of an assessment like TIMSS 2007, since they are vital in ensuring that the survey covers the appropriate range of cognitive skills across the content domains already outlined.

The first domain, *knowing,* covers the facts, procedures, and concepts students need to know, while the second, *applying,* focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The third domain, *reasoning*, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems.

These three cognitive domains are used for both grades, but the balance of testing time differs, reflecting the difference in age and experience of students in the two grades. For fourth and eighth grades, each content domain will include items developed to address each of the three cognitive domains. For example, the number domain will include knowing, applying, and reasoning items as will the other content domains.

Exhibit 5 shows the target percentages of testing time devoted to each cognitive domain for both the fourth- and eighth-grade assessments.

Exhibit 5: **Target Percentages of the TIMSS 2007 Mathematics Assessment Devoted to Cognitive Domains at Fourth and Eighth Grades**

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

Knowing

Facility in using mathematics, or reasoning about mathematical situations, depends on mathematical knowledge and familiarity with mathematical concepts. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she has understood, the greater the potential for engaging a wide range of problem-solving situations and for developing mathematical understanding.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the factual knowledge that provides the basic language of mathematics, and the essential mathematical facts and properties that form the foundation for mathematical thought.

Procedures form a bridge between more basic knowledge and the use of mathematics for solving routine problems, especially those encountered by many people in their daily lives. In essence a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Knowledge of concepts enables students to make connections between elements of knowledge that, at best, would otherwise be retained as isolated facts. It allows them to make extensions beyond their existing knowledge, judge the validity of mathematical statements and methods, and create mathematical representations.

This cognitive domain covers the following behaviors:

1.	**Recall**	Recall definitions; terminology; number properties; geometric properties; and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).

2.	**Recognize**	Recognize mathematical objects, shapes, numbers and expressions. Recognize mathematical entities that are mathematically equivalent (e.g., equivalent familiar fractions, decimals and percents; different orientations of simple geometric figures).
3.	**Compute**	Carry out algorithmic procedures for $+, -, \times, \div$, or a combination of these with whole numbers, fractions, decimals and integers. Approximate numbers to estimate computations. Carry out routine algebraic procedures.
4.	**Retrieve**	Retrieve information from graphs, tables or other sources; read simple scales.
5.	**Measure**	Use measuring instruments; use units of measurement appropriately; and estimate measures.
6.	**Classify/Order**	Classify/group objects, shapes, numbers and expressions according to common properties; make correct decisions about class membership; and order numbers and objects by attributes.

Applying

Problem solving is a central aim, and often means, of teaching school mathematics, and hence this and supporting skills (e.g., select, represent, model) feature prominently in the domain of *applying* knowledge and conceptual understanding. In items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations and solve problems. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations are fundamental to success in the subject.

The problem settings are more routine than those aligned with the reasoning domain. The routine problems will typically have been standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems will have been in words that set the problem situation in a quasi-real context. Though they range in difficulty, each of these types of "textbook" problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned procedures.

Problems may be set in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. Therefore, problem solving is included not only in the *applying* domain, with emphasis on the more familiar and routine tasks but also in the *reasoning*, domain.

This cognitive domain covers the following behaviors:

1.	**Select**	Select an efficient/appropriate operation, method or strategy for solving problems where there is a known algorithm or method of solution.
2.	**Represent**	Display mathematical information and data in diagrams, tables, charts, or graphs, and generate equivalent representations for a given mathematical entity or relationship.
3.	**Model**	Generate an appropriate model, such as an equation or diagram for solving a routine problem.
4.	**Implement**	Follow and execute a set of mathematical instructions. Given specifications, draw figures and shapes.

| 5. | **Solve Routine Problems** | Solve routine problems (i.e., problems similar to those target students are likely to have encountered in class). For example, use geometric properties to solve problems. Compare and match different representations of data (eighth grade) and use data from charts, tables, graphs, and maps to solve routine problems. |

Reasoning

Reasoning mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned. Non-routine problems may be purely mathematical or may have real-life settings. Both types of items involve transfer of knowledge and skills to new situations, and interactions among reasoning skills are usually a feature. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation or because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

Even though of the many behaviors listed within the reasoning domain are those that may be drawn on in thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

This cognitive domain covers the following behaviors:

1.	**Analyze**	Determine and describe or use relationships between variables or objects in mathematical situations; use proportional reasoning (fourth grade); decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; visualize transformations of three-dimensional figures; compare and match different representations of the same data (fourth grade); and make valid inferences from given information.
2.	**Generalize**	Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.
3.	**Synthesize/ Integrate**	Combine (various) mathematical procedures to establish results, and combine results to produce a further result. Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas.
4.	**Justify**	Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.
5.	**Solve Non-routine Problems**	Solve problems set in mathematical or real life contexts where target students are unlikely to have encountered closely similar items, and apply mathematical procedures in unfamiliar or complex contexts. Use geometric properties to solve non-routine problems.